Curved Space-time in Matrix Models

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Superstring theory still contains a lot of unknown aspects including its nonperturbative dynamics. In order to elucidate these aspects, we have to gain a deep understanding of superstring theory from various directions. In this thesis, we focus on the gauge/gravity duality, which gives the gauge theory description of superstring, and the matrix models, which were proposed as a nonperturbative formulation of superstring theory. We show that various kinds of curved space-times and topological invariants defined on them can be described by matrix models. This realization of curved space-time is based on the combination of the construction of fuzzy space and Taylor's matrix T-duality for gauge theories on D-branes. This result does not only shed light on the matrix model formulation of superstring theory, but also gives us a new method of analysing the gauge/gravity duality in the strong 't Hooft coupling regime as follows. In terms of the above realization of curved spaces, we first reveal the relationships between theories with $SU(2|4)$ symmetry which include SYM on $R \times S^3/Z_k$, SYM on $R \times S^2$ and the plane wave matrix model (PWMM). We show that each of SYM on $R \times S^3/Z_k$ and SYM on $R \times S^2$ can be realized as a certain large N limit of PWMM. We also check that these relations are consistent with the the classical solutions in the gravity side which was obtained by Lin and Maldacena. Based on these relations, we propose a nonperturbative regularization of $\mathcal{N} = 4$ SYM on $R \times S^3$ in terms of PWMM. This regualrization is achieved by considering the large N reduction on a finite volume instead of the matrix T-duality. This regularization preserves the gauge symmetry and $SU(2|4)$ symmetry manifestly. We also test the validity of this regularization by calculating some correlation functions, the vacuum expectation value of the circular BPS Wilson loop operator and the free energy. We find that all the results agree with those in $\mathcal{N} = 4$ SYM on $R \times S^3$.