Measurement of g-factor of Cosmic Muon

Nanjo Lab B4
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Introduction

Motivation : We wanted to study something related to spin.

→ magnetic moment of muon is described as $\hat{\mu} = -g\dfrac{e\hat{\mathbf{S}}}{2m_\mu}$

Our goal is measuring g factor !

$m_\mu \simeq 105.7\ [\mathrm{Mev/c^2}]$
Spin $= \hbar/2$
$g \simeq 2$

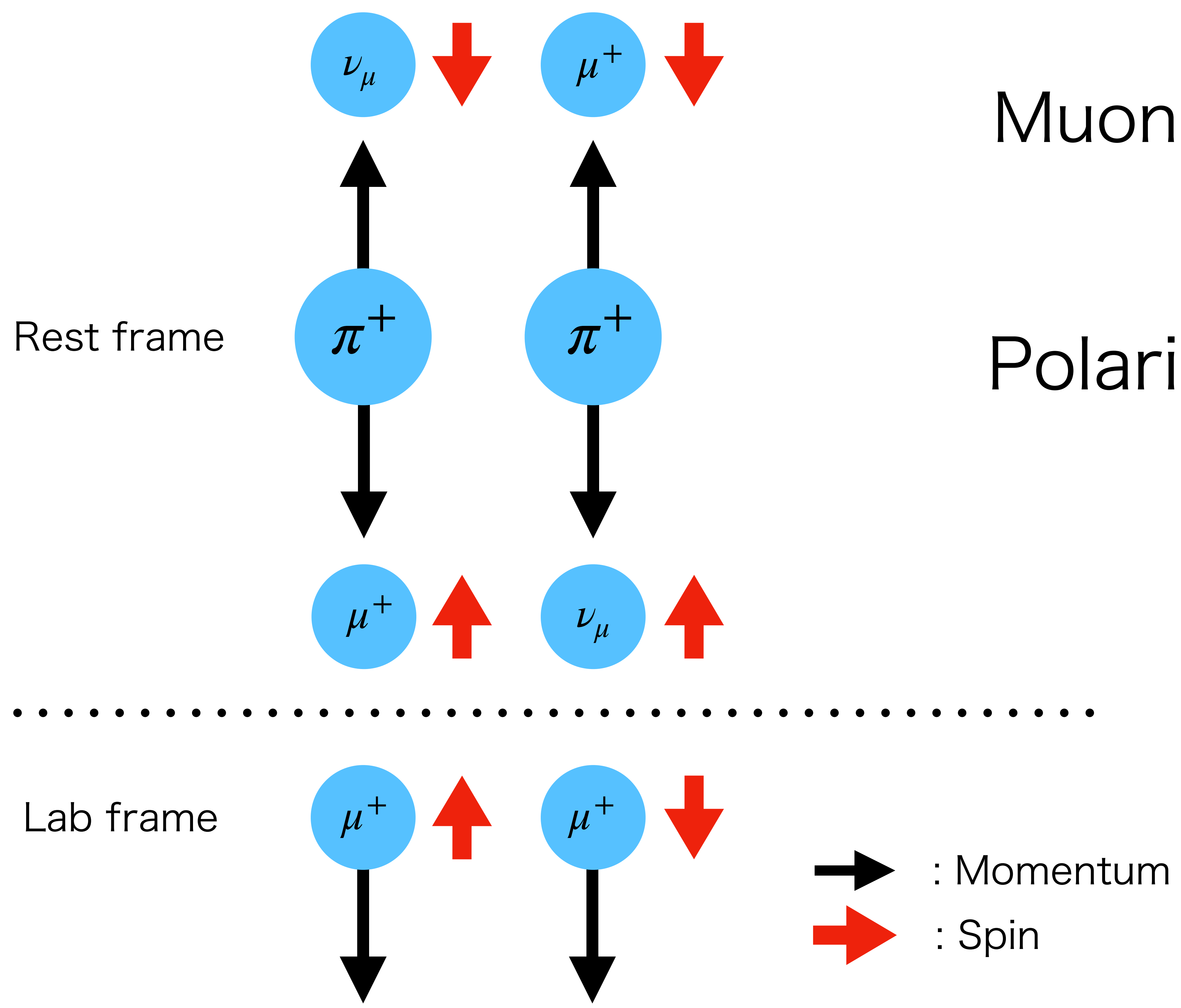
Polarization of cosmic muon

Cosmic muon is generated by the decay of pion.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

Energy spectrum of pion is $\pi(\gamma)\, d\gamma \propto \gamma^{-\alpha}\, d\gamma \quad (\alpha : 2.6 \sim 2.7, \gamma = E_\pi/m_\pi)$

Muon is polarized $99.59\,\%\ (CL90\%)$ in the pion's rest frame

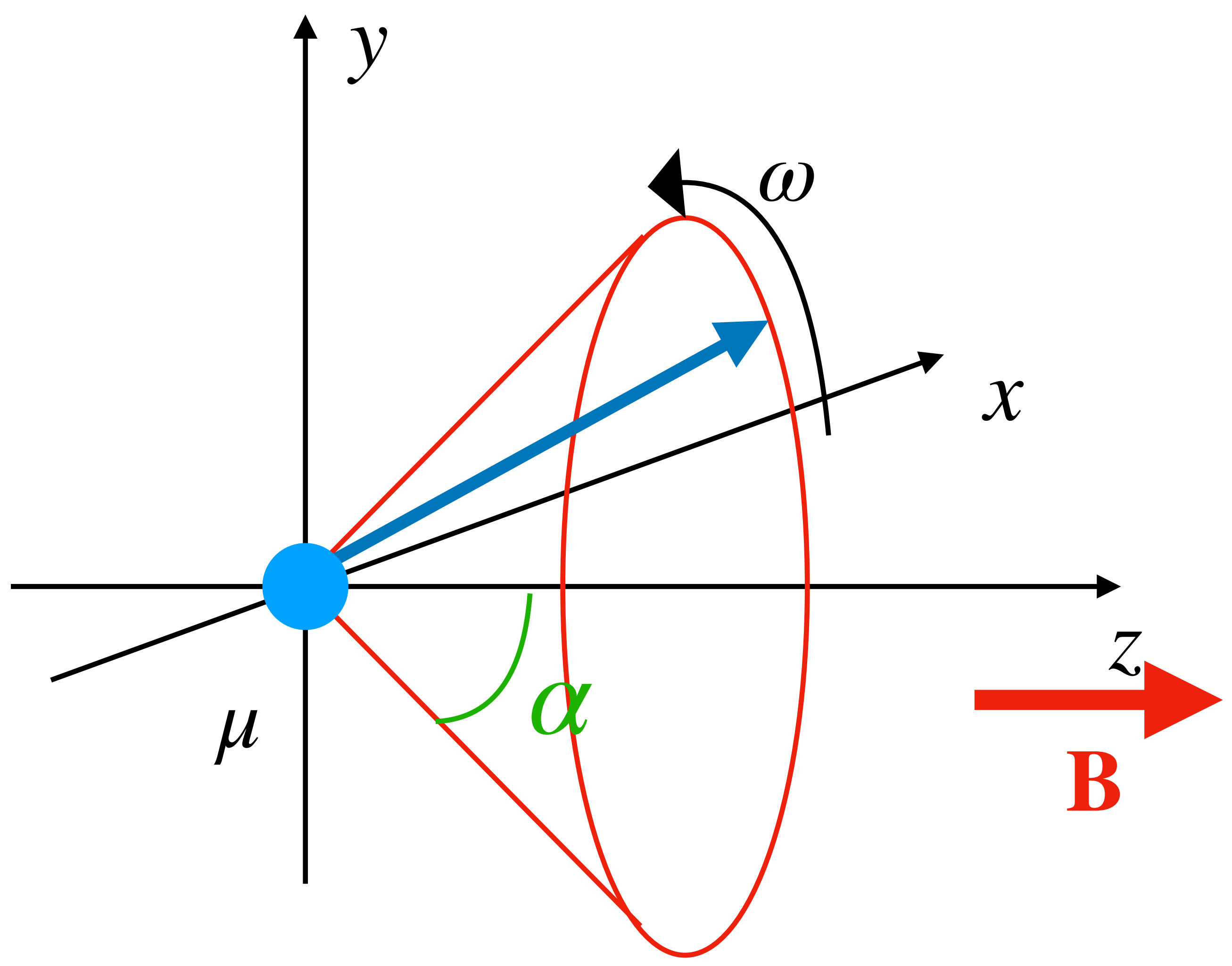


Polarization $P_0 = (\alpha/3)\beta\beta^* \quad (\beta = v/c)$ (* means pion's rest frame)

Substitute $\alpha = 2.7, \quad \beta \simeq 1, \quad \beta^* = 0.27$

$$\rightarrow P_0 = 0.24$$

Behavior of spin in magnetic field



In magnetic field,

Hamiltonian is given as $\hat{\mathscr{H}} = -\hat{\mu} \cdot \hat{\mathbf{B}}$

Then, the expected value of spin state is as follows.

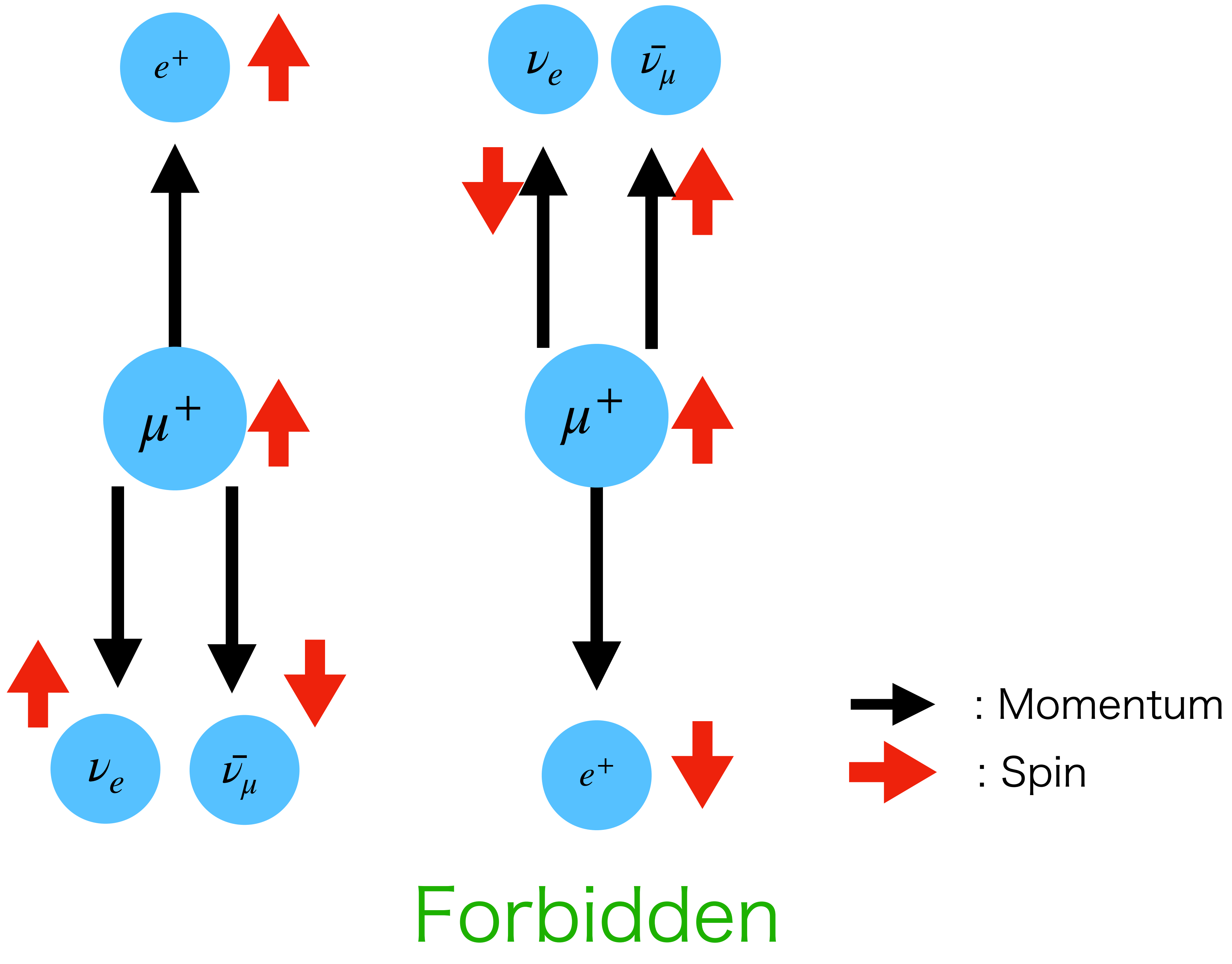
$$\langle S_x \rangle = \frac{\hbar}{2} \sin\alpha \cos\omega t, \quad \langle S_y \rangle = \frac{\hbar}{2} \sin\alpha \sin\omega t, \quad \langle S_z \rangle = \frac{\hbar}{2} \cos\alpha$$

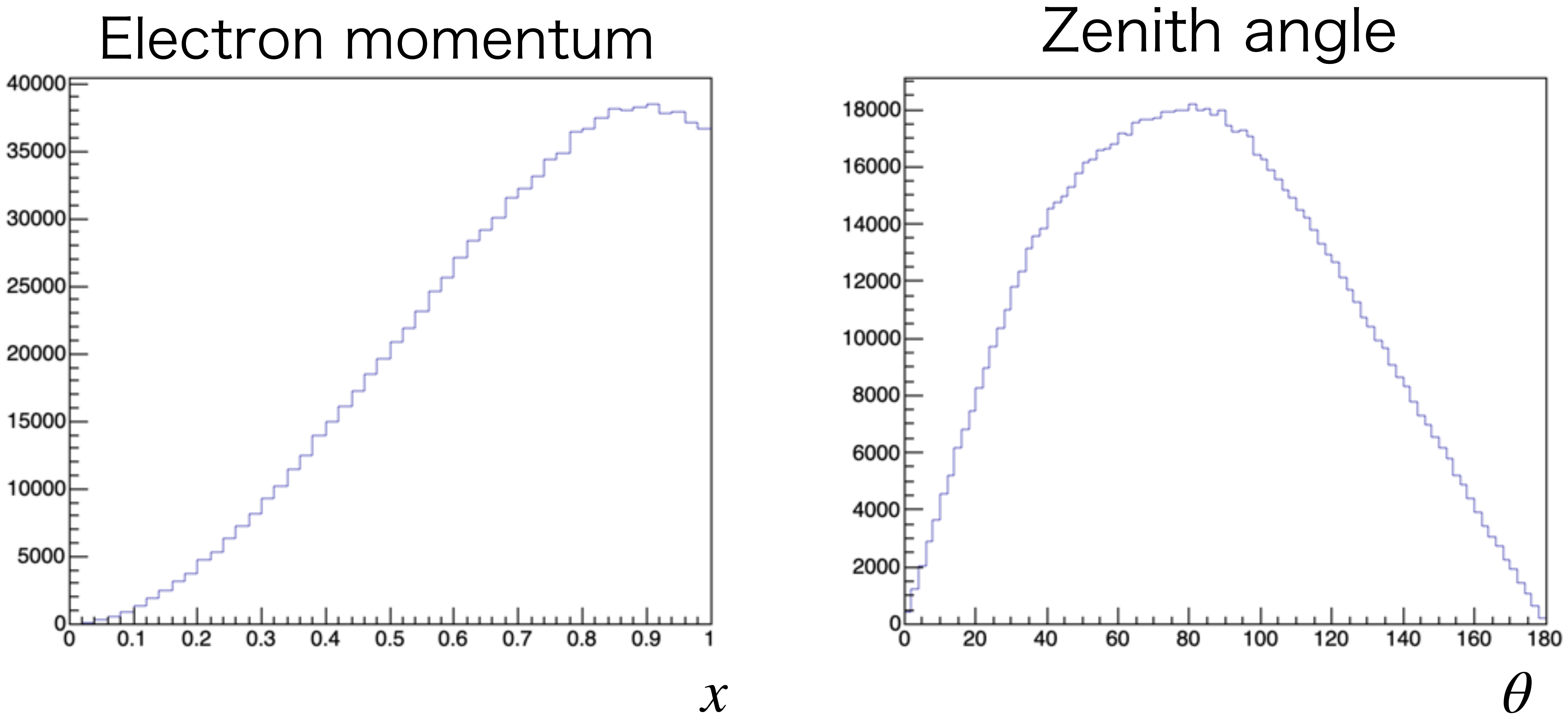
, where $\omega = \frac{geB}{2m}$

→ Spin precess around the direction of magnetic field with angular velocity ω

Decay of muon

$$\mu^+ \rightarrow e^+ + \overline{\nu_\mu} + \nu_e$$





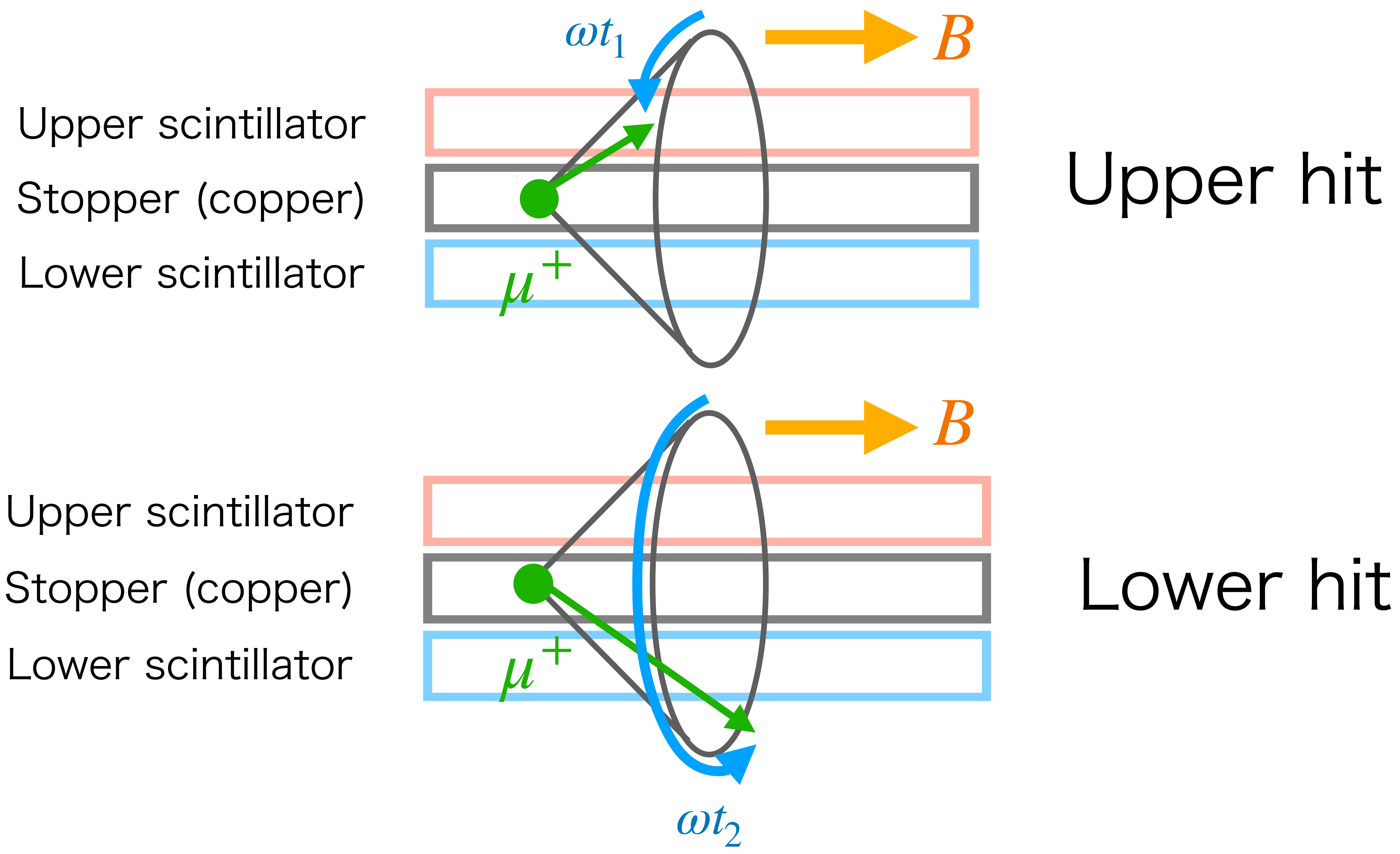
$$\frac{d^2\Gamma}{dxd\theta} \propto \{3 - 2x + cos\theta \cdot (2x - 1)\} \cdot sin\theta \cdot x^2$$

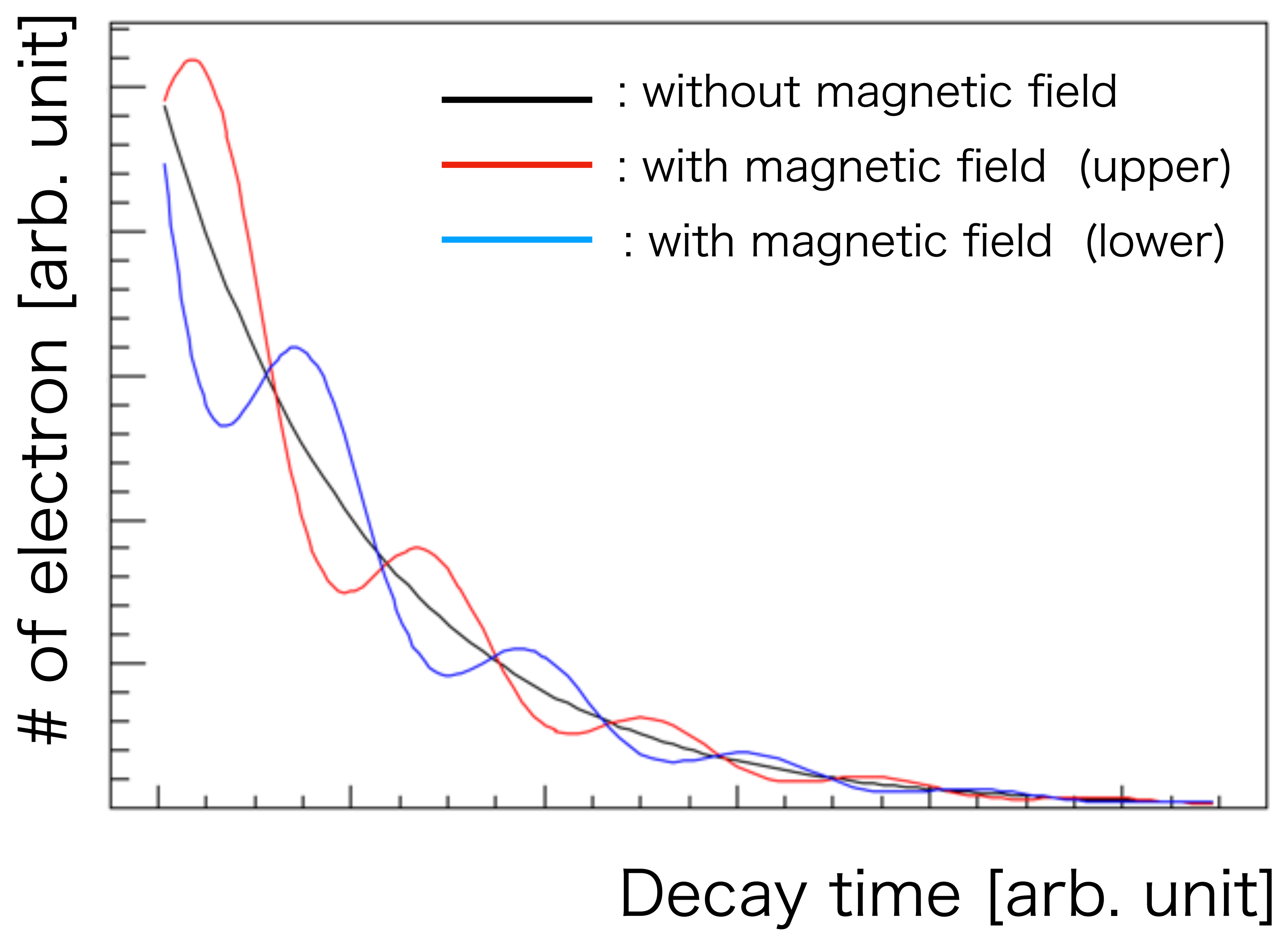
θ : angle between the electron momentum and muon spin

x : $E_e/max(E_e)$ where $max(E_e) = (m_\mu^2 + m_e^2)/2m_\mu$

$\rightarrow e^+$ tend to be emitted in the direction of μ^+ spin

Detection of emitted electron





With magnetic field, decay curve oscillate with period $2\pi/\omega$.

Then, the equation of decay curve is given as $N(t) = A\exp(-t/\tau)(1 + B\cos(\omega t + C))$

→ by fitting, g factor is obtained !

Detector design

- In our previous experiment, about 100 electrons are detected by the $15\mathrm{cm} \times 6\mathrm{cm}$ detector
- Expecting events = $100 \times 33 \times 2 \times 4 \simeq 20{,}000$

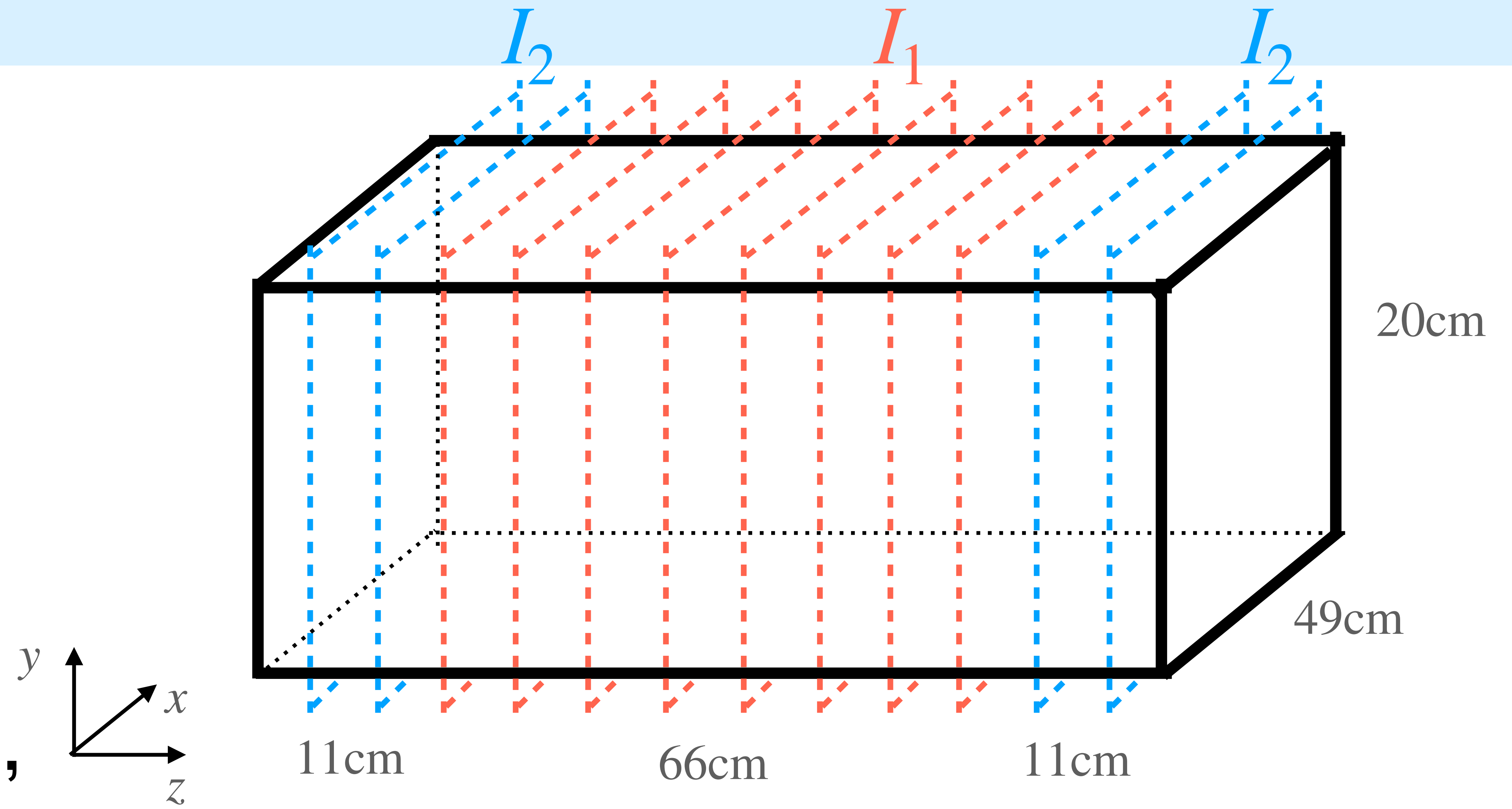
33: Increasing triger area

2: Detecting both sides of cupper

4: Increasing acceptance

Optimizing currents

The magnetic field should be flat in the muon-stopping area $6.5\text{cm} \le z \le 81.5\text{cm}$.



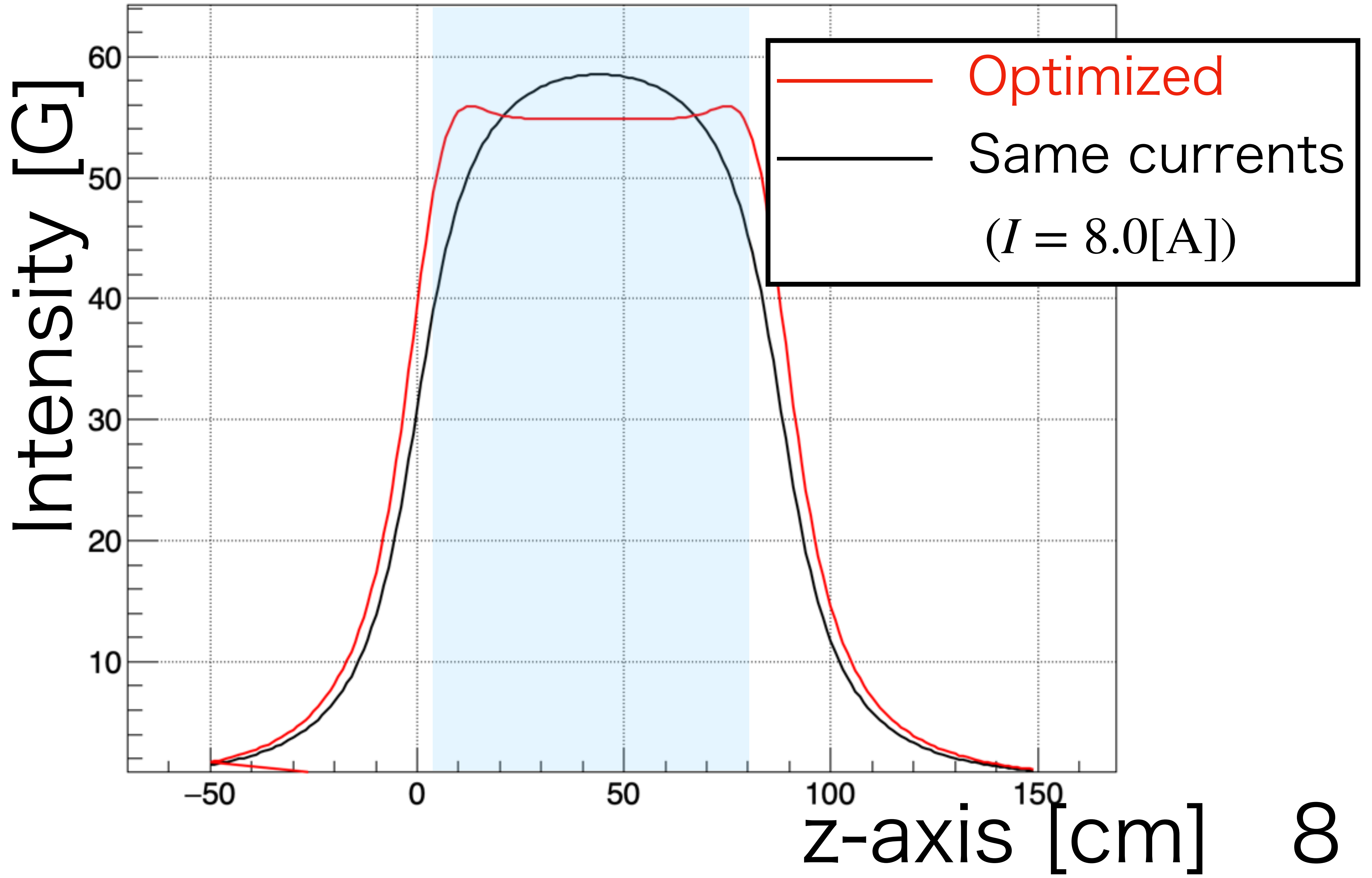
Currents are determined to minimize the deviation from the desired magnetic field B^*,

$$\sigma^2 = \sum_i (B(\mathbf{r_i};\, I_1, I_2) - B^*)^2.$$

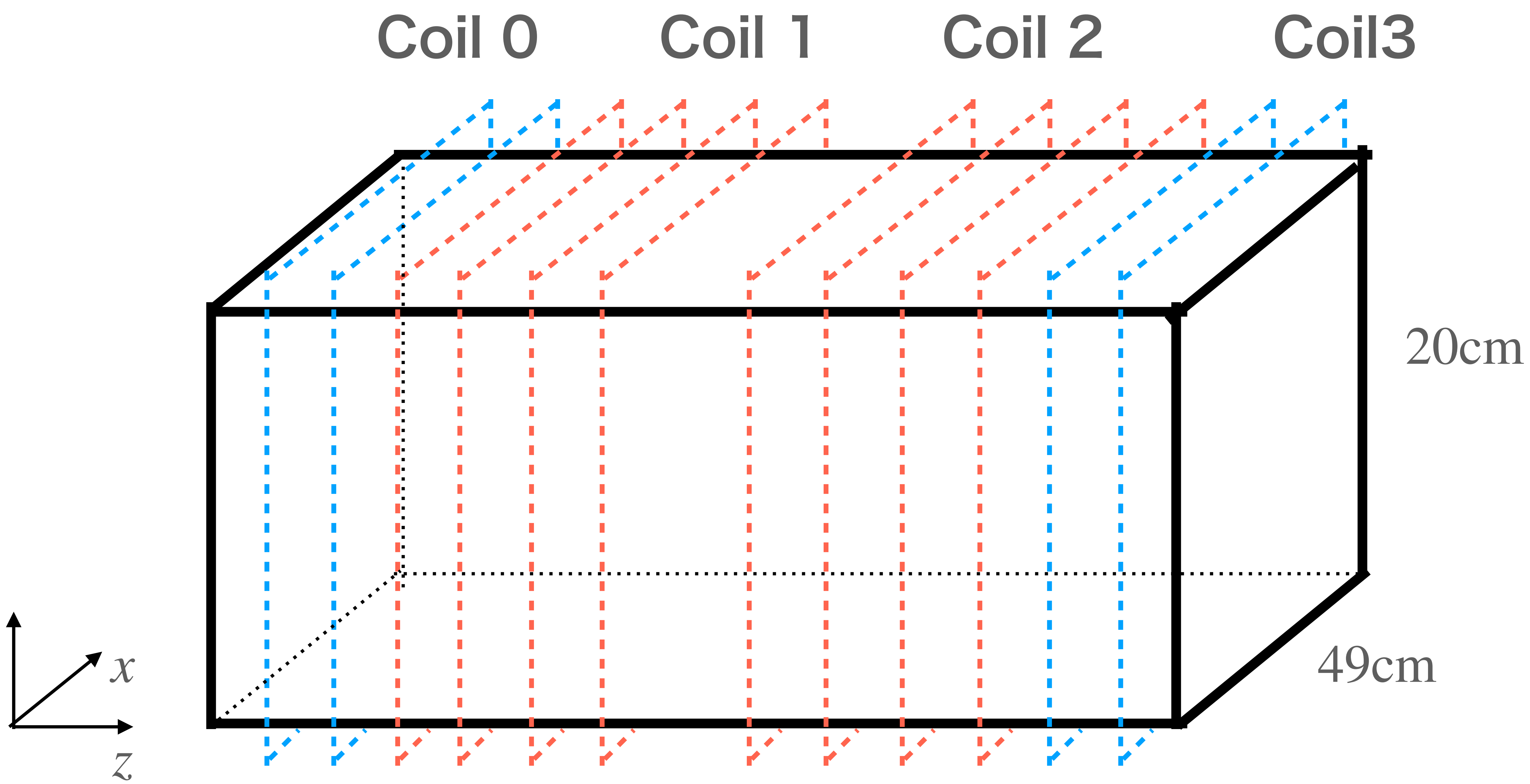
Considering the limits of power of supplies,

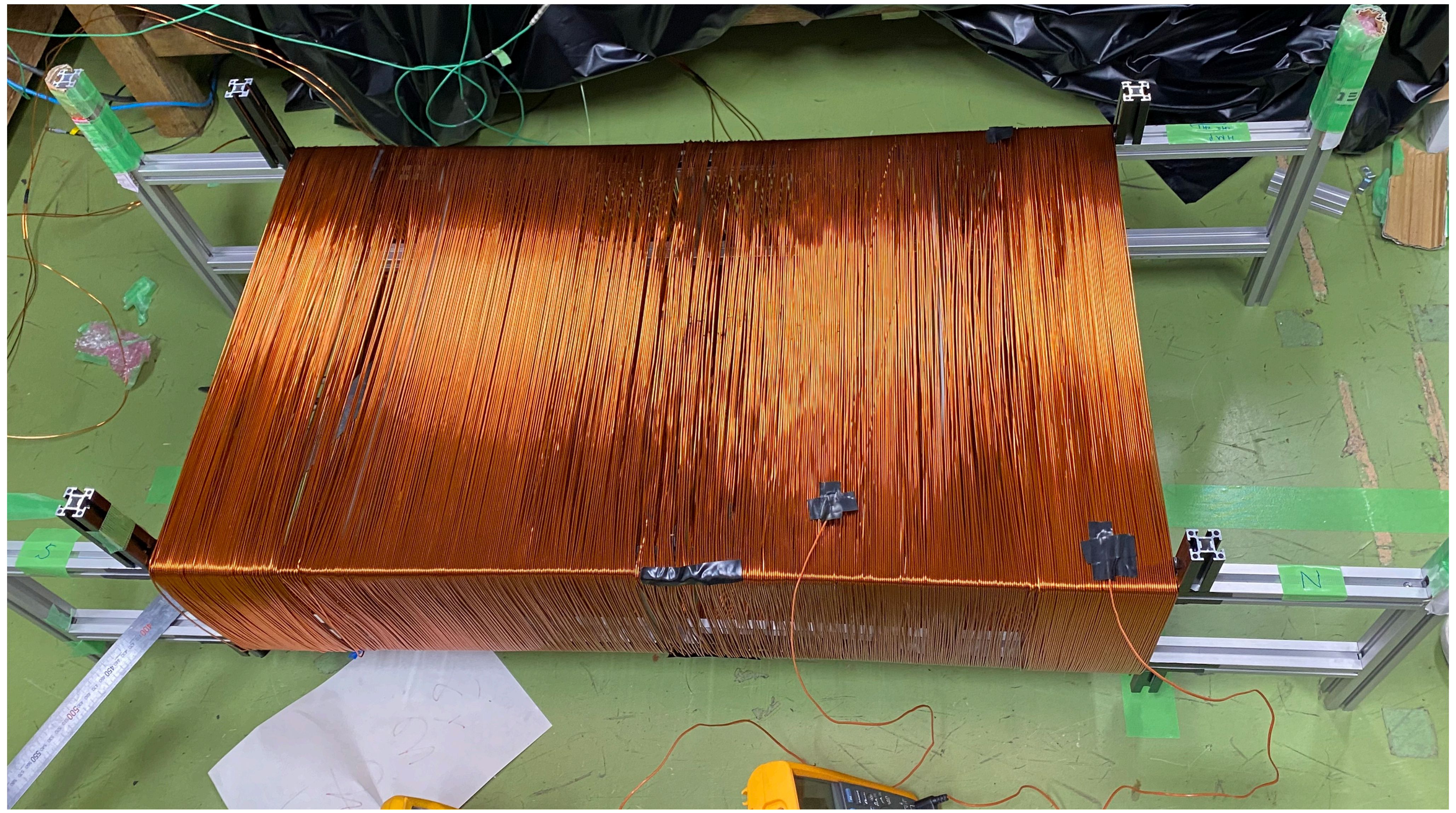
$$\rightarrow I_1 = 7.3[\text{A}], \quad I_2 = 12.1[\text{A}]$$

and $B^* = 55[\text{G}]$.



Making the coil

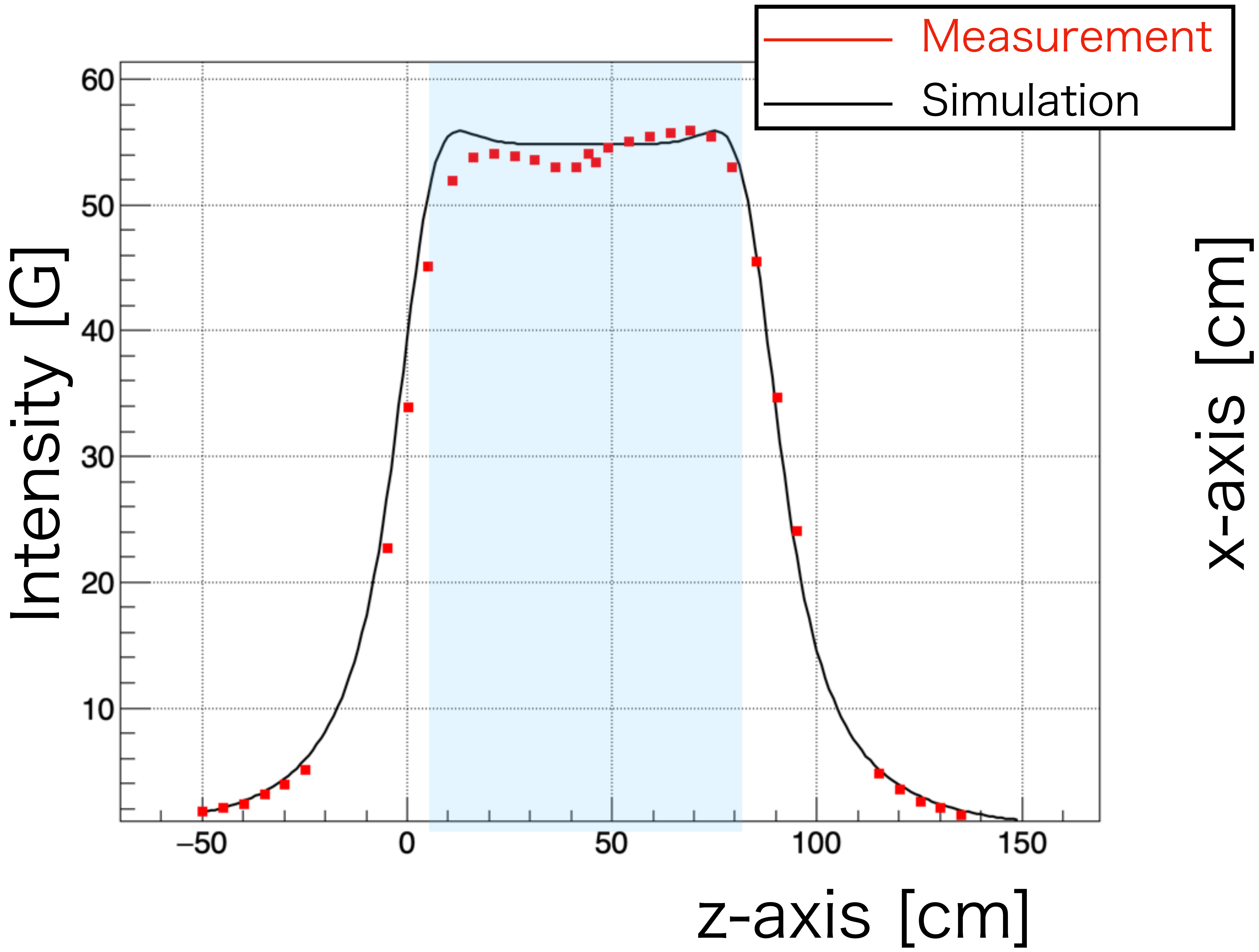


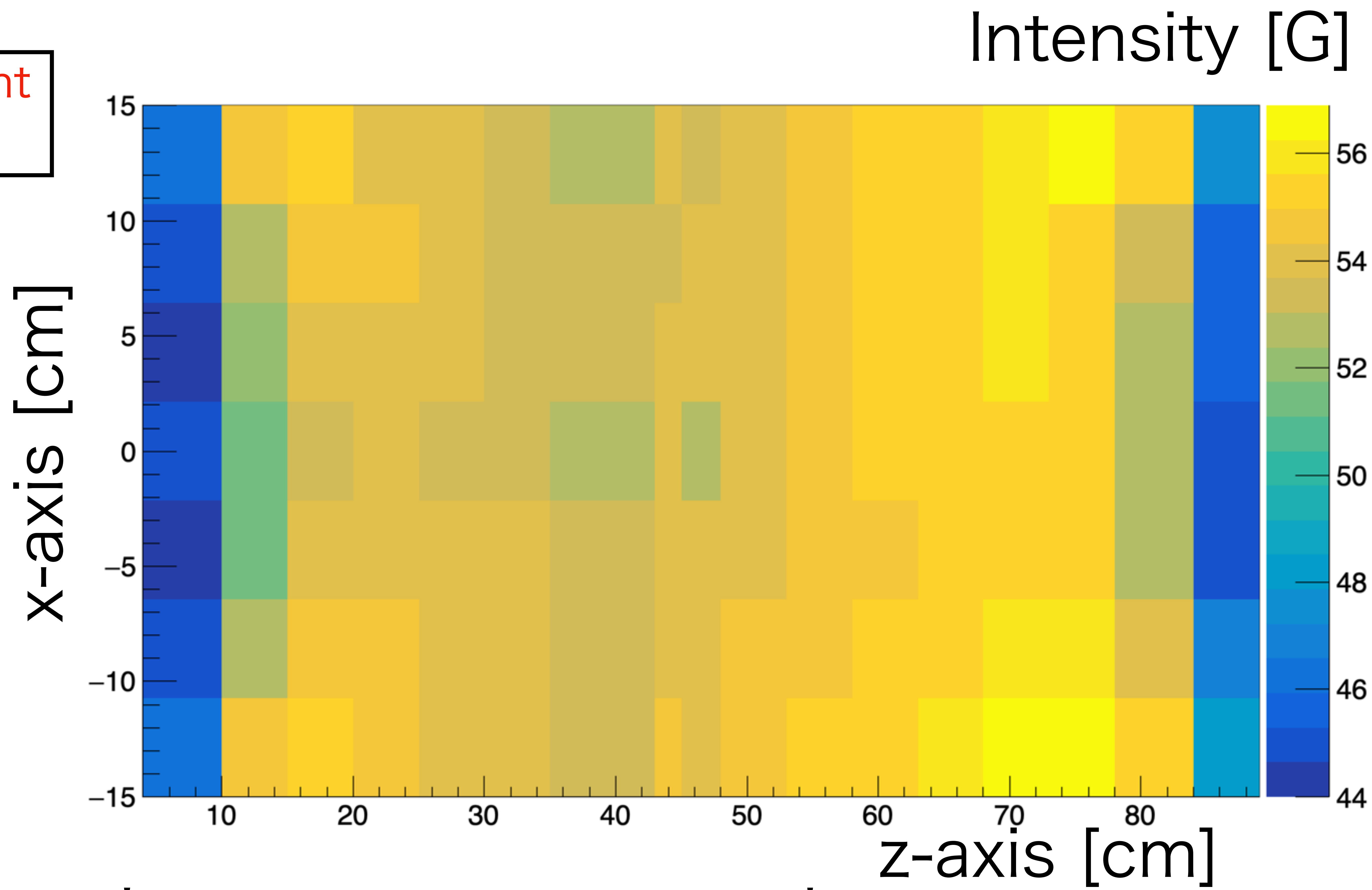


	Coil0	Coil1	Coil2	Coil3
Length	11cm	33cm	33cm	11cm
Loop	**3**	1	1	**2**
Current	**4.03A**	7.28A	7.28A	**6.09A**
Power	41.8W	139.2W	139.9W	62.7W

Instead of 12.1[A], increasing the loops and changing the currents of coil0 and coil3.

Measurement of Magnetic field



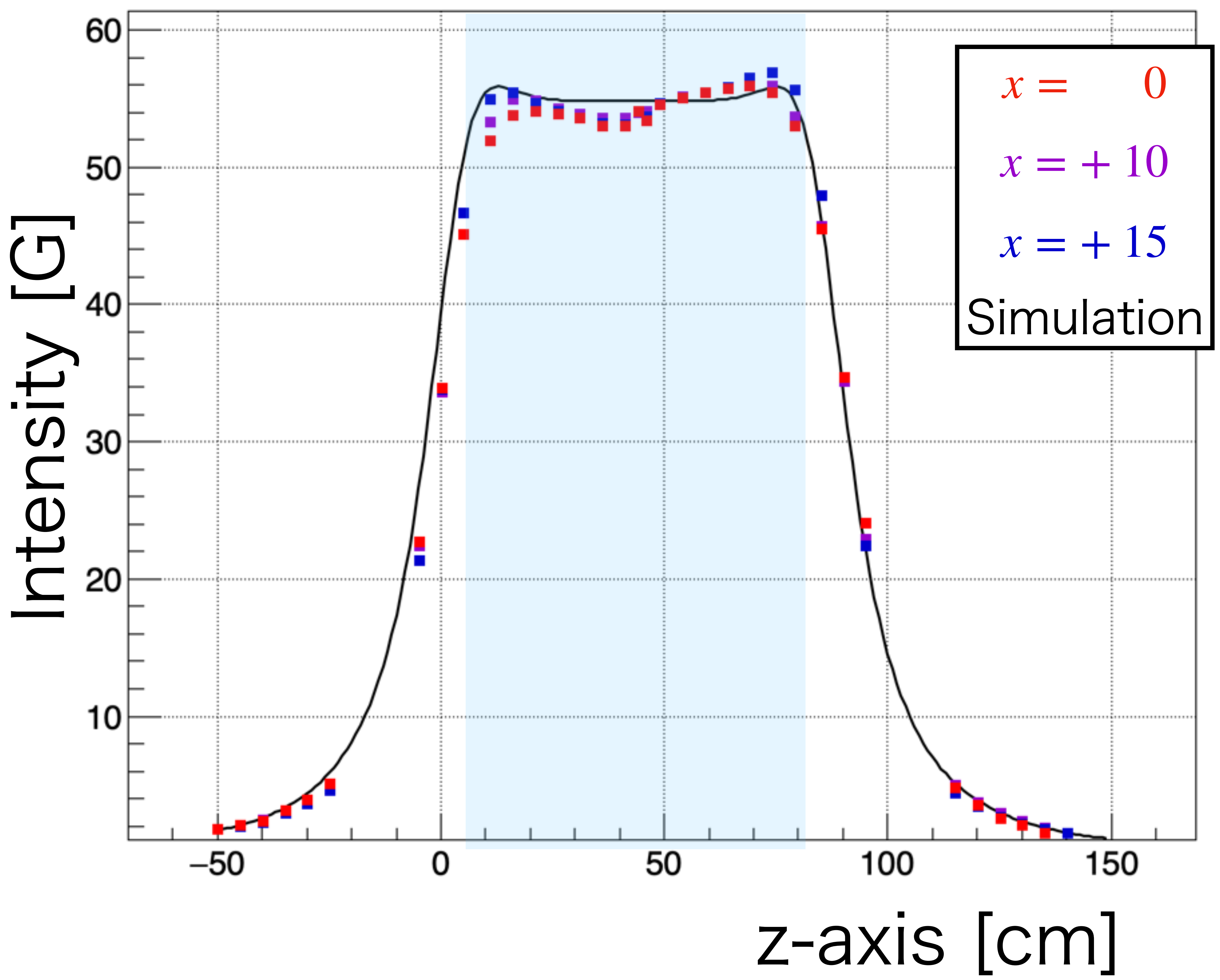


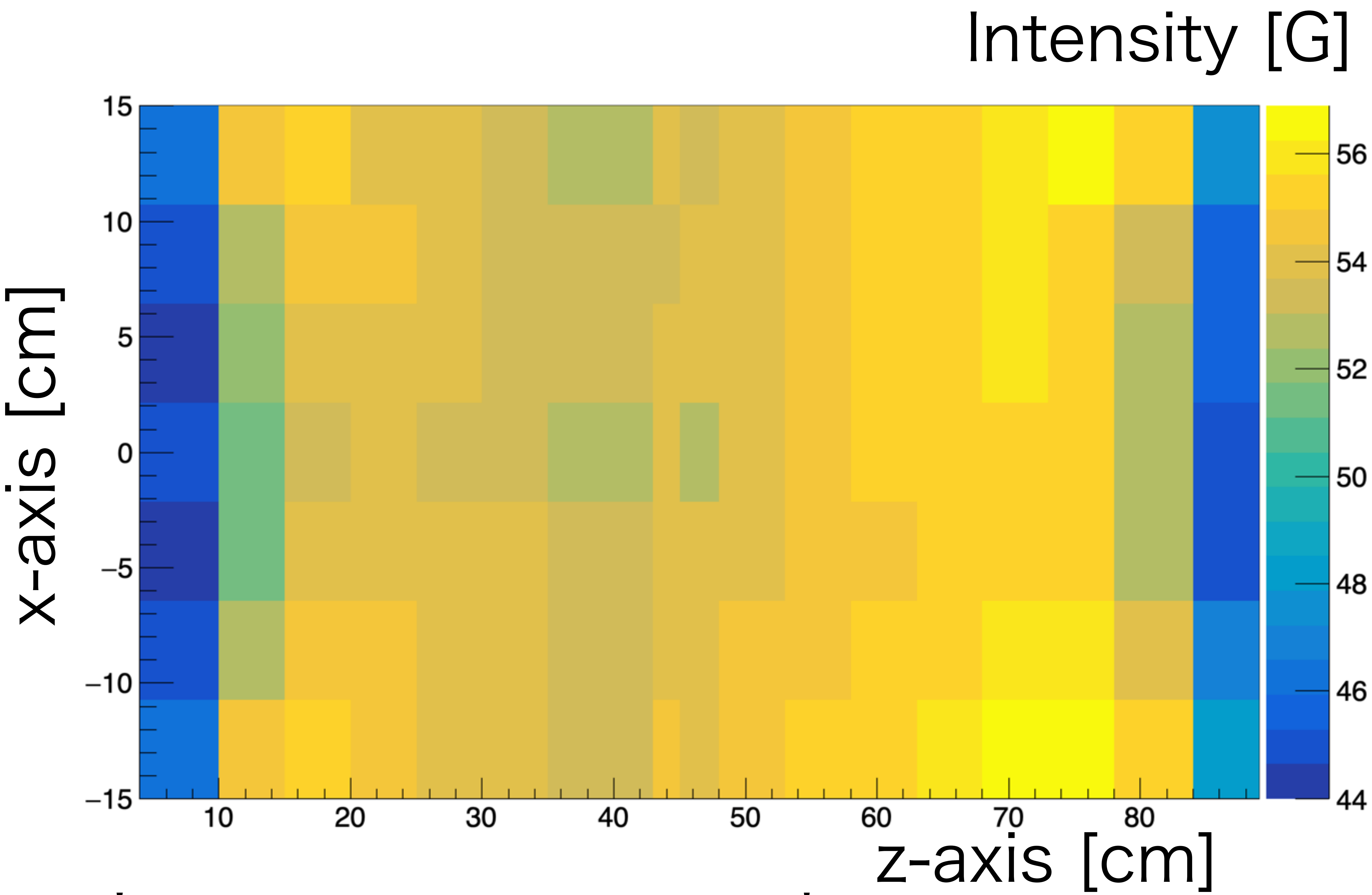
In $6.5\mathrm{cm} \leq z \leq 81.5\mathrm{cm}$, the mean:

$B = 54.2[\mathrm{G}]$

→The uniformity is 1.9%.

Measurement of Magnetic field





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$B = 54.2[\text{G}]$

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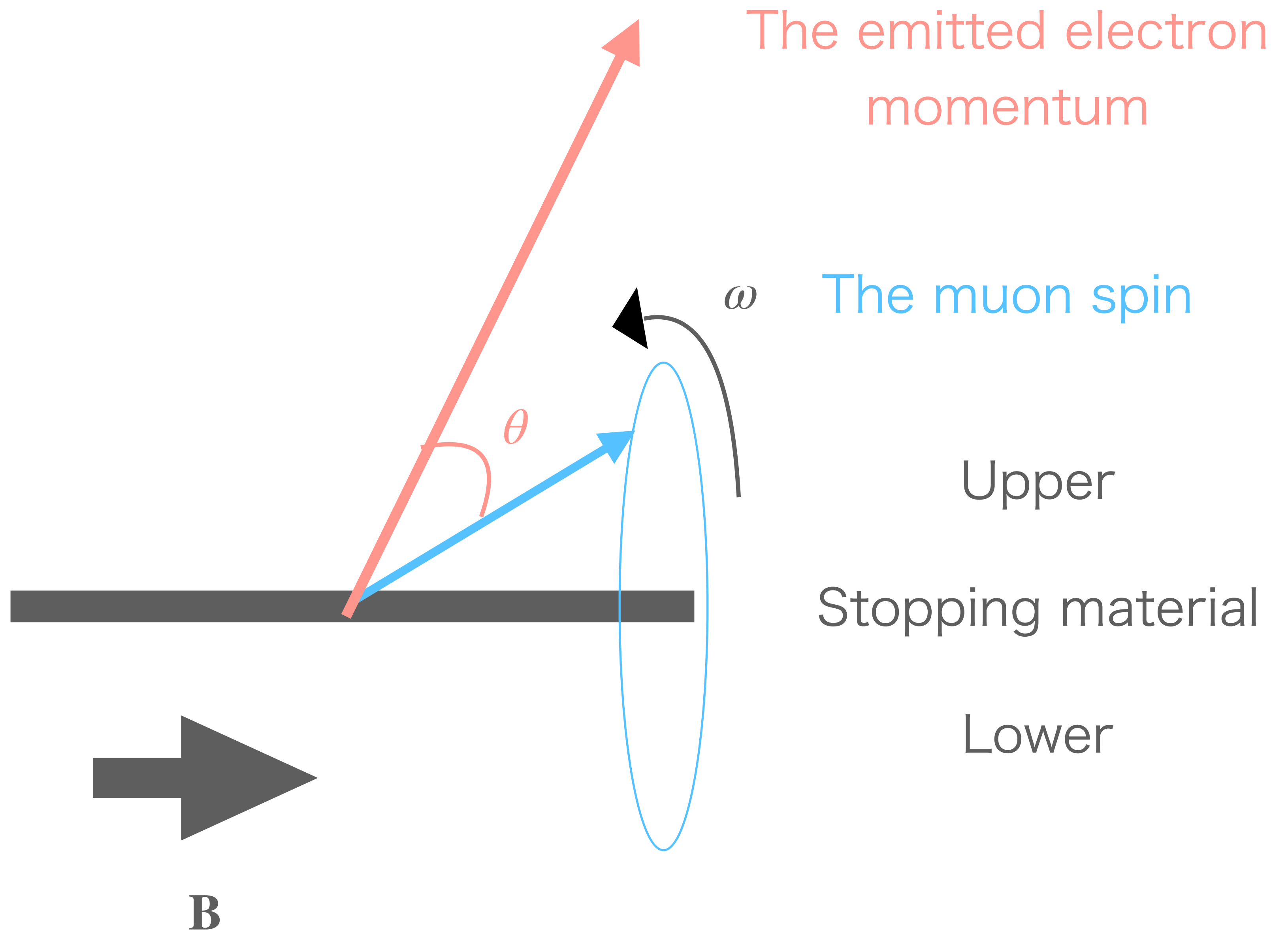
Simulation for Magnetic field

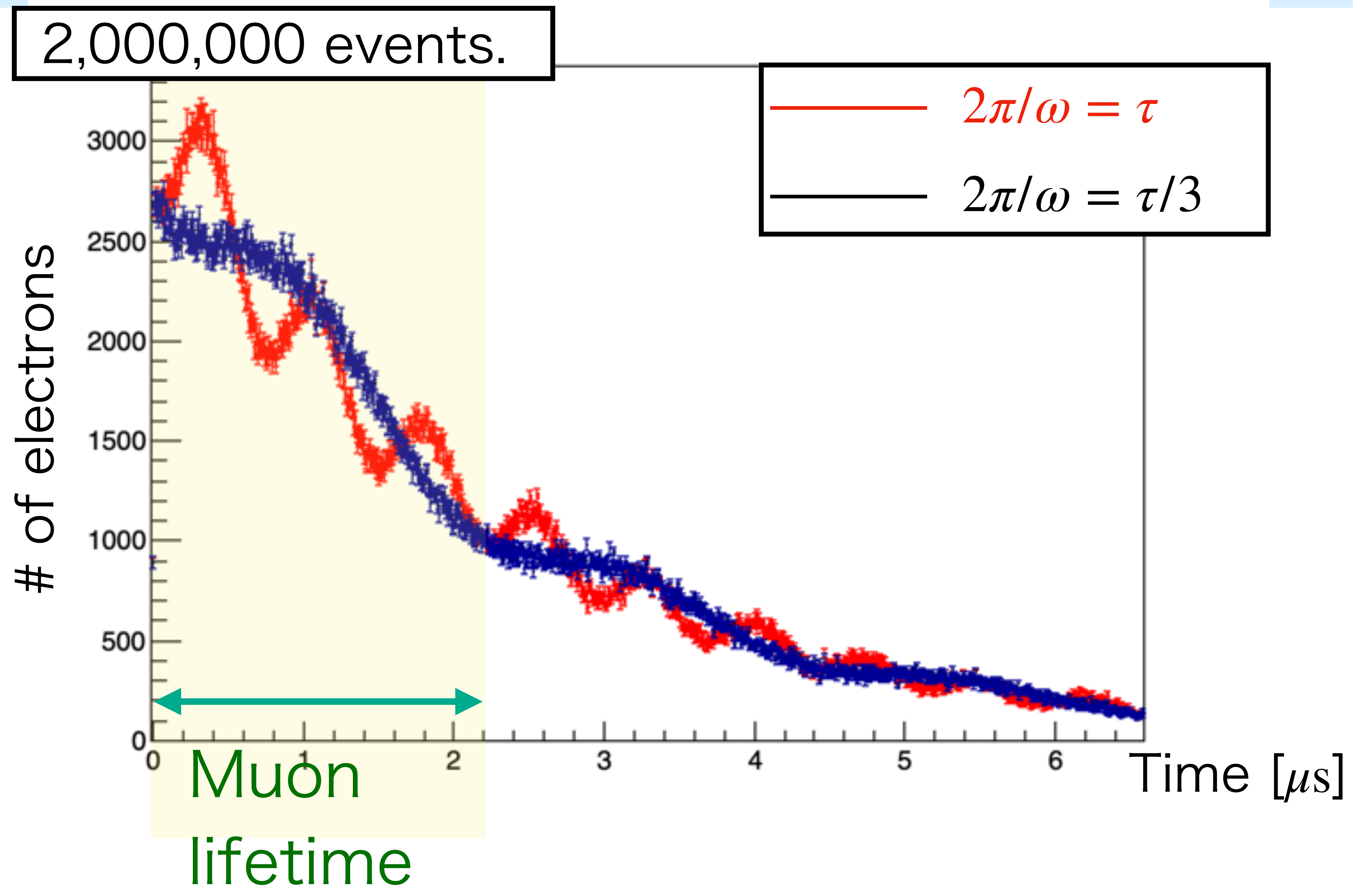
The magnetic field we made is best? → evaluate using MC.

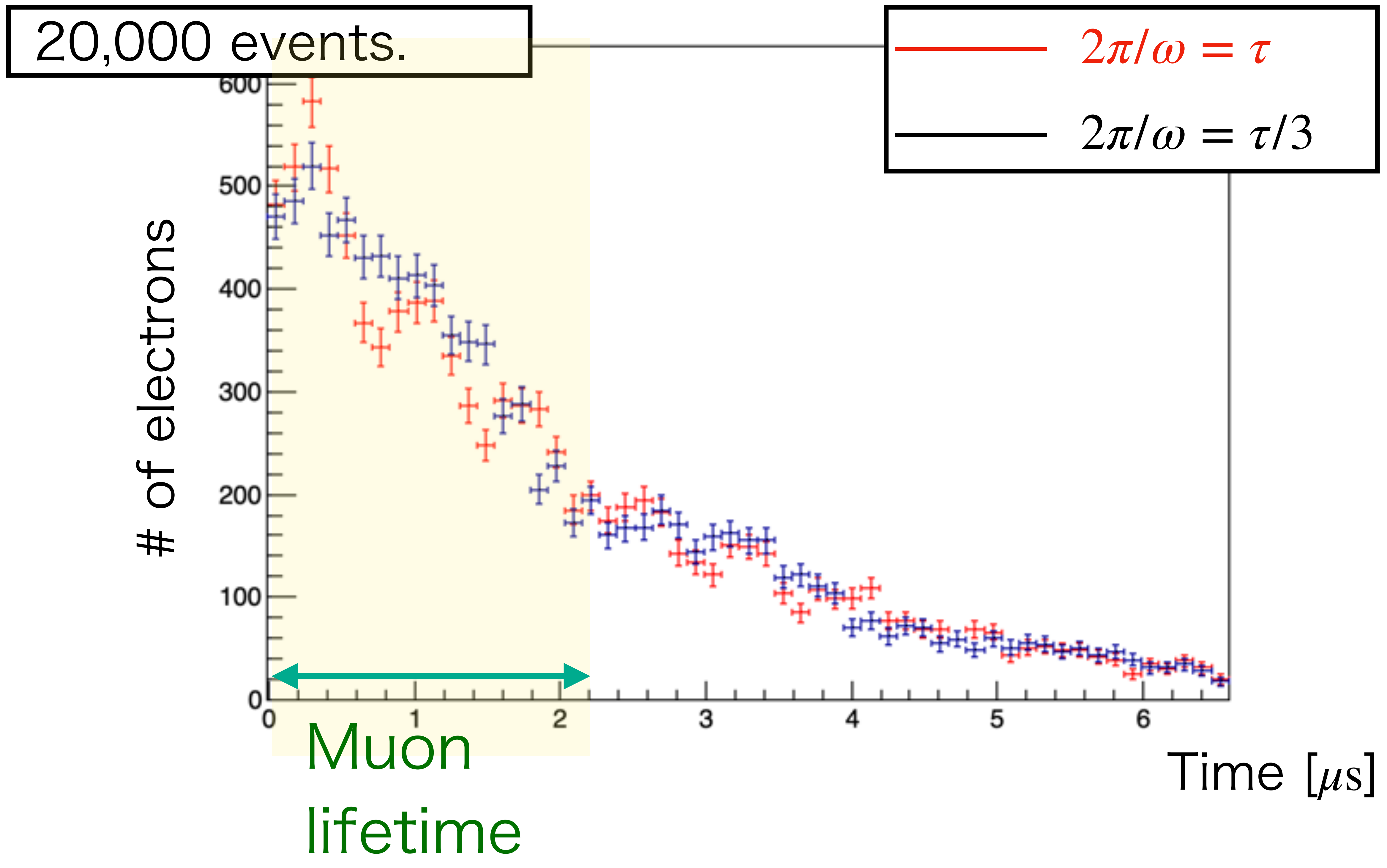
Conditions:

- The zenith angle distribution of muons is $\cos^2\theta$.
- 100% polarized.
- The direction of emitted electrons is only considered.
  →Upper or Lower to stopping material.
- The conditions of acceptance and energy are ignored.
- 20,000 electrons can be detected.



Result of Simulation





Fit the result by the function $N(t) = A\exp(-t/\tau)(1 + B\cos(\omega t + C))$.

Lifetime τ and frequency ω are common to upper and lower detectors.

The larger the frequency, the more clearly the oscillation appears, but more bins are needed.

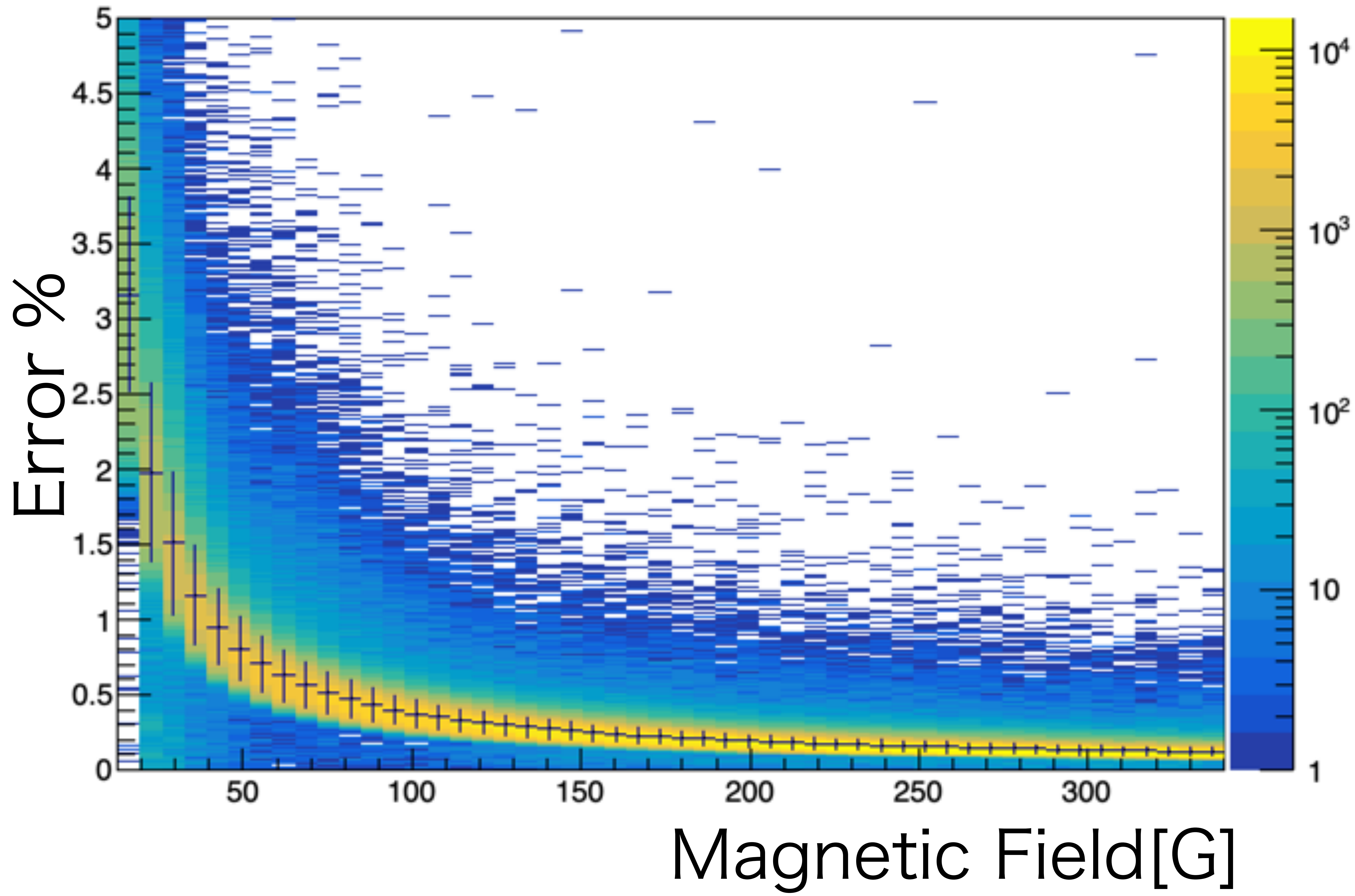
Optimization of Magnetic field

The fit results of the simulation are given $\omega_i \pm \sigma_i$.

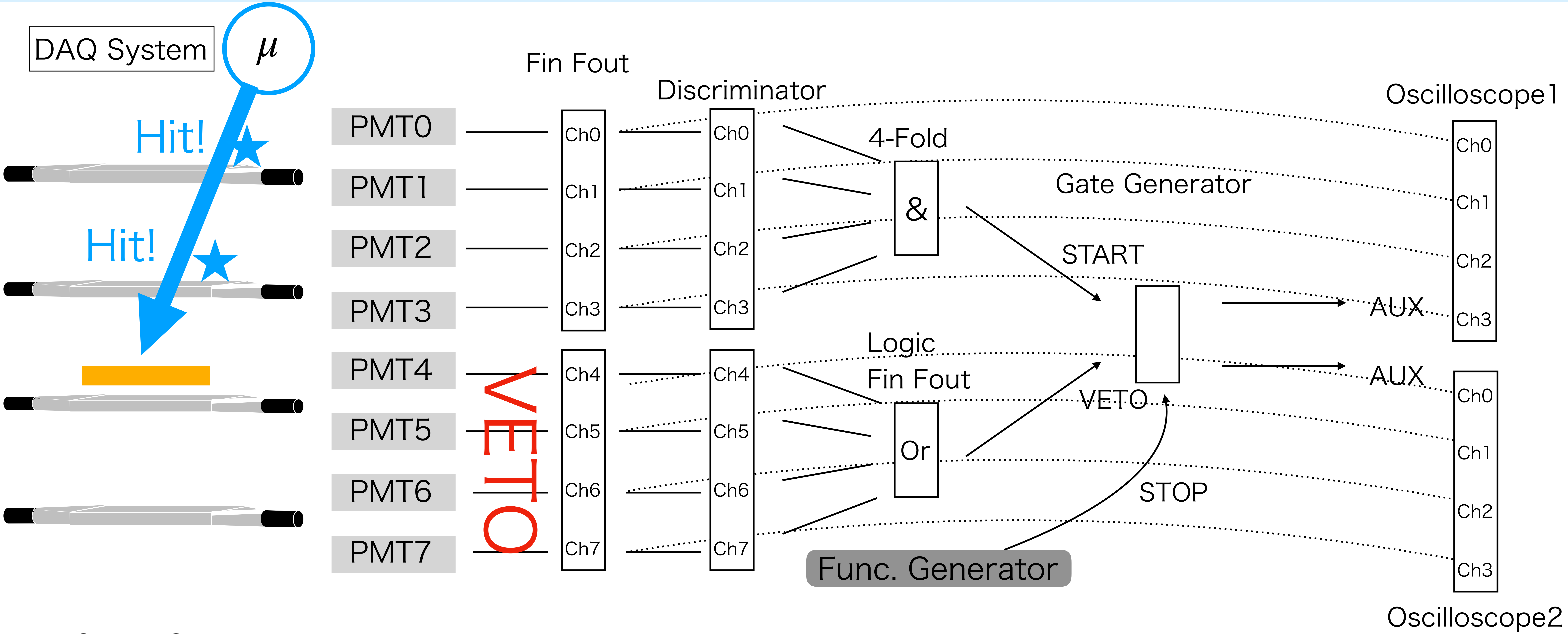
That simulation is executed repeatedly, and each execution gets the fitting error σ_i / ω_i.

If 200,000 entries, more stronger magnetic field is better.

However, the fitting error may be below 1% in $B^* = 55[\mathrm{G}]$.



Data Acquisition (DAQ)



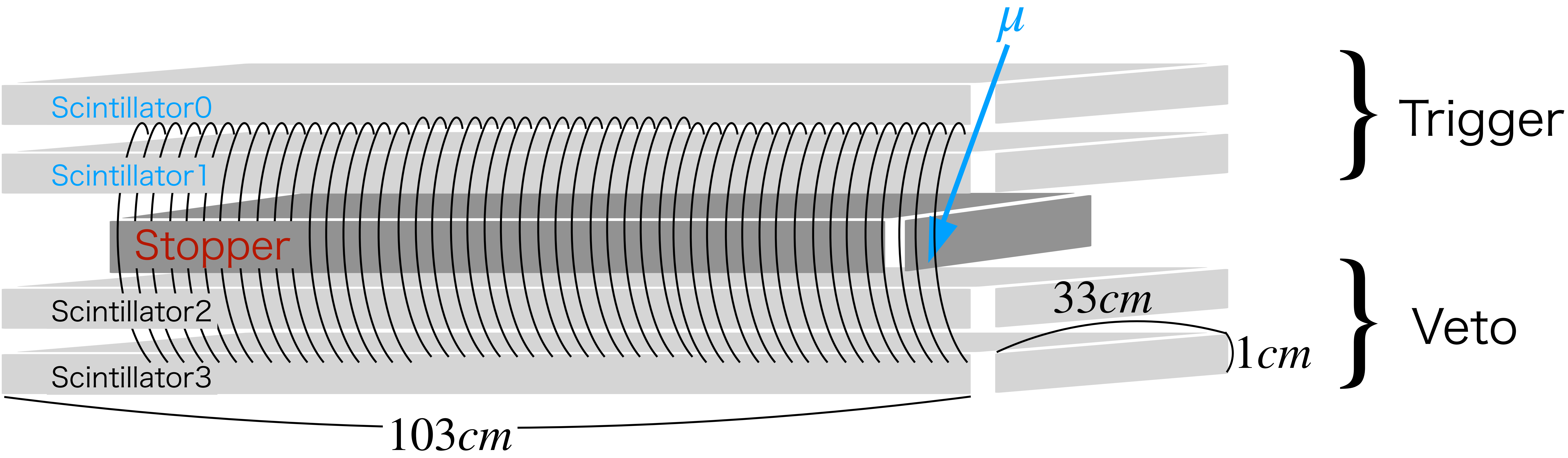
- Our Oscilloscope takes data using trigger signal from AUX.

→ To synchronize two Oscilloscope, we must create same AUX signals.
Realized using LATCH mode & Function Generator.

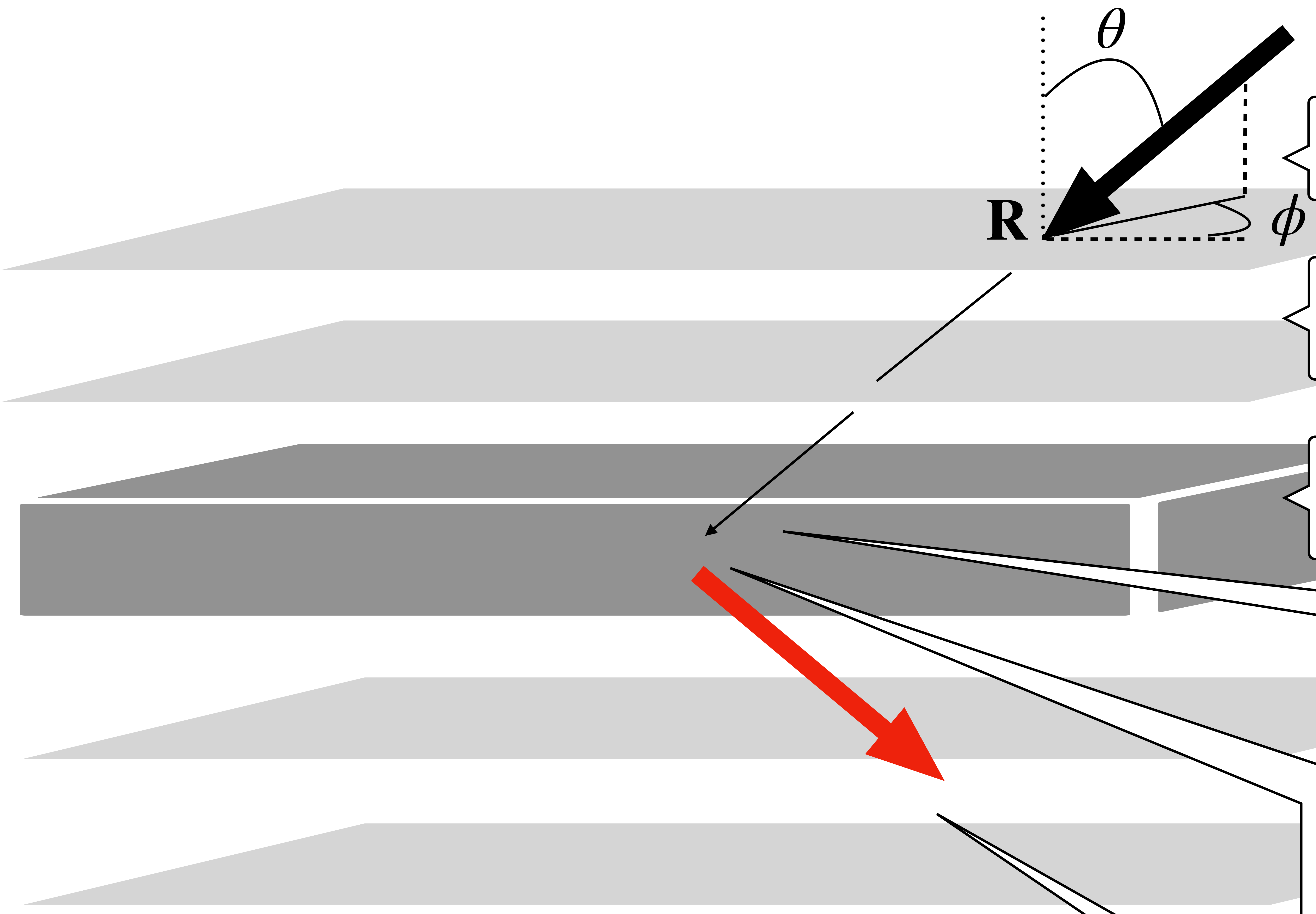
Setup optimization

GOAL : Estimate #expected electrons from muon decay with the ideal Setup using simulation.

1. Optimize the Scintillator layout (accounting DAQ Limit).
2. Optimize the stopper layout (copper).
3. Estimate # of expected electrons from muon decay.



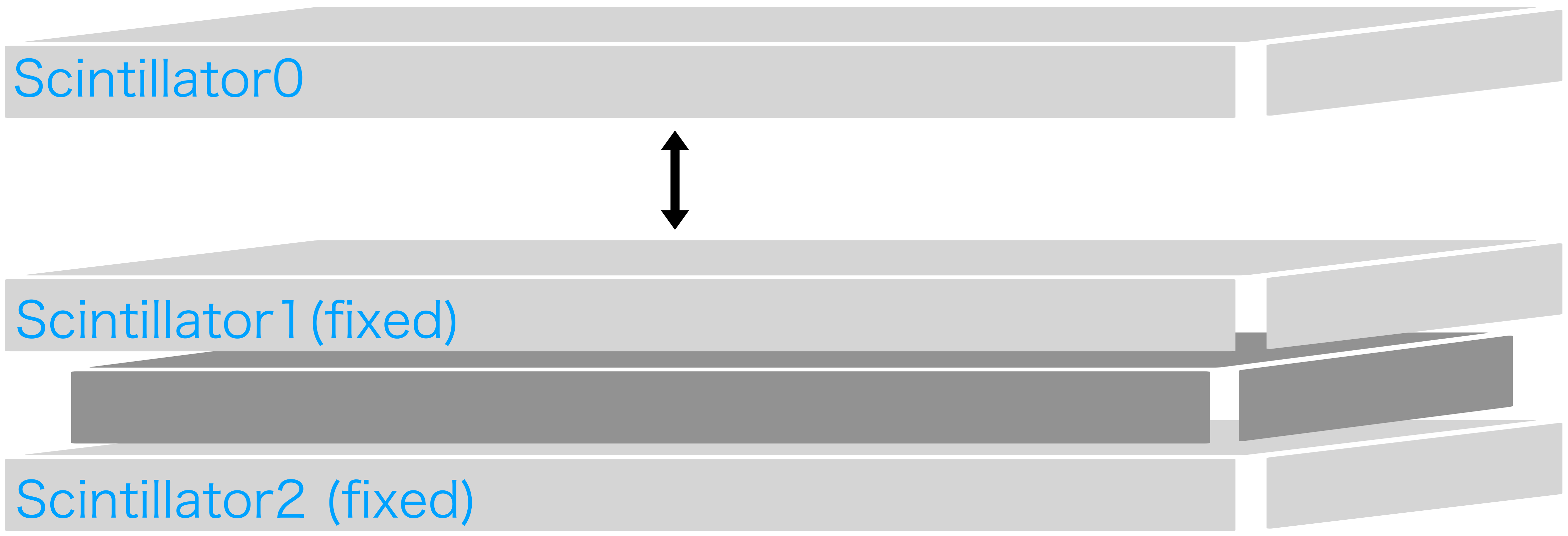
Flow of the Simulation

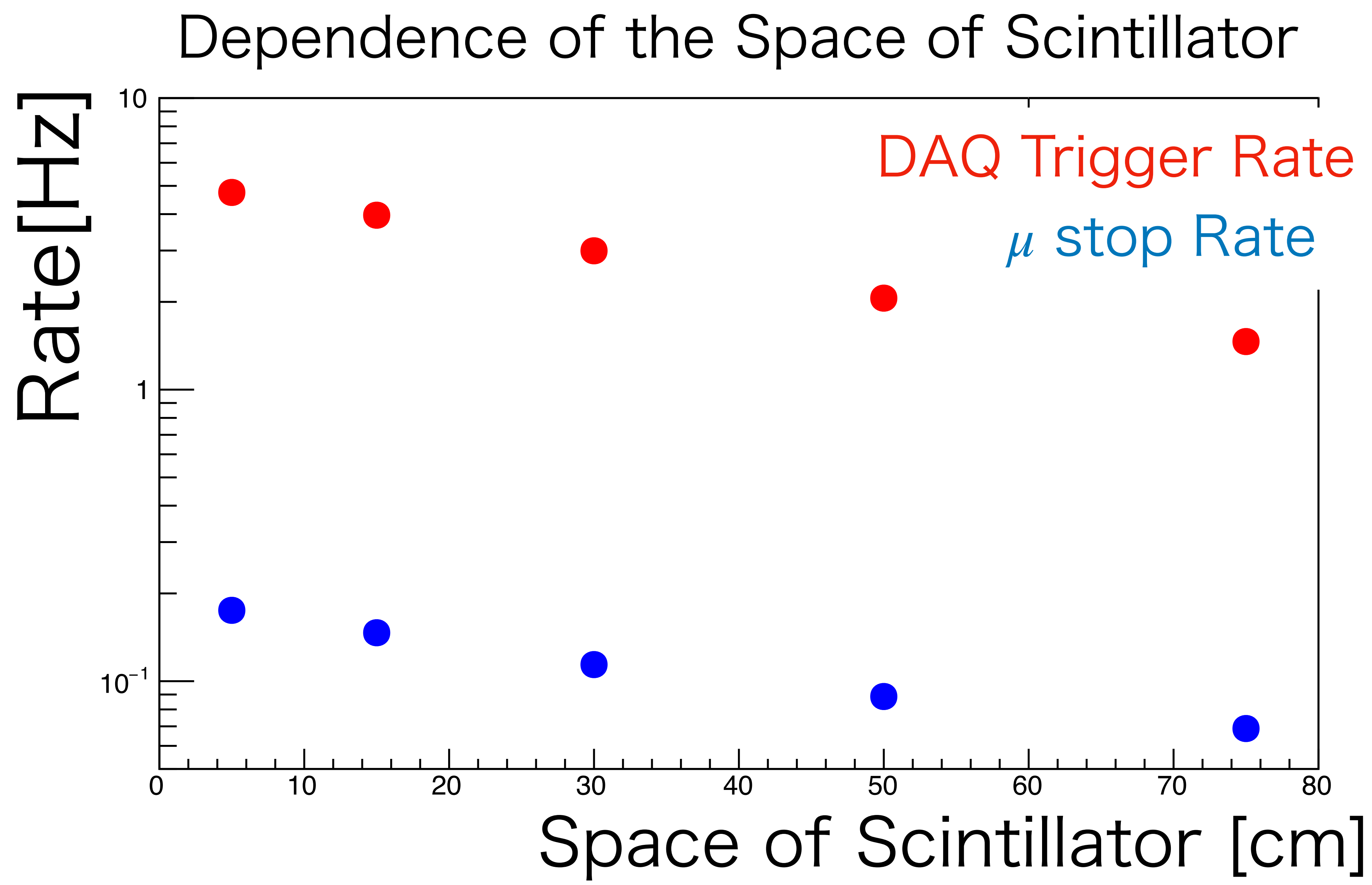


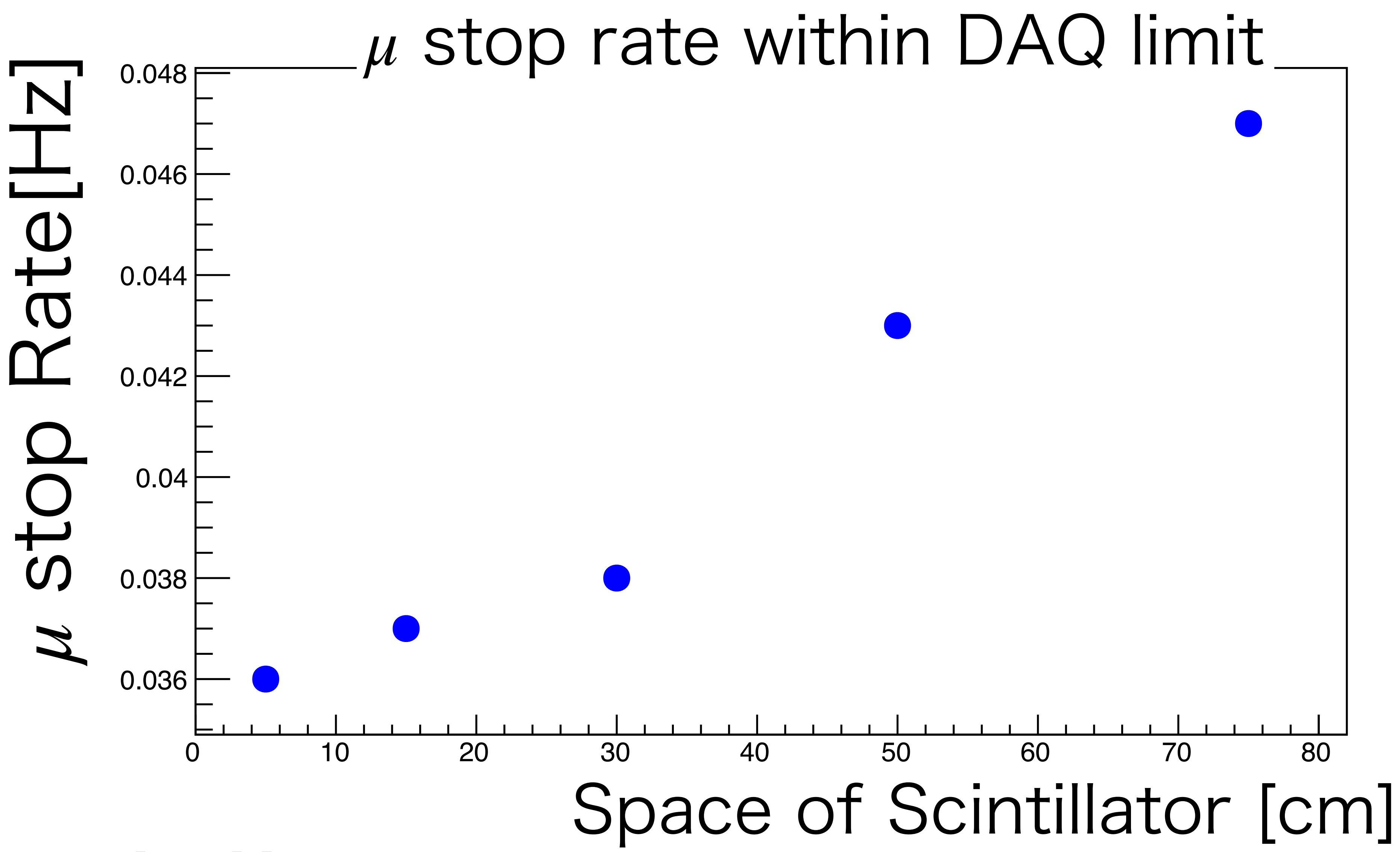
1. Determine E_μ, θ_μ, ϕ_μ,$\mathbf{R}_\mu$.
2. Check if muons enter this scintillator.
3. Check if muons stops inside of stopper.
4. Cut half of muons.
5. If stops, Rotates muon spin And determine E_e, θ_e, ϕ_e.
6. Then, check if electrons enters two scintillators.

Spacing dependence

Checked the spacing dependence.





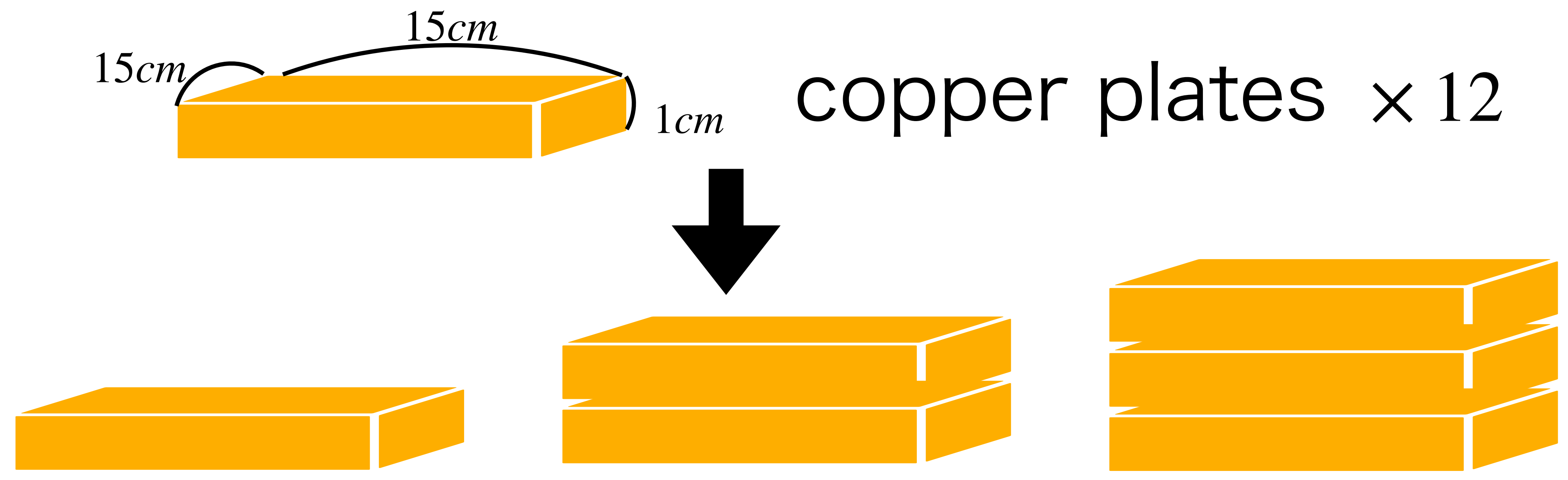


- The narrower the space is, the more # of muons
- However, DAQ limit is 1Hz, calculated ● $\times \frac{1\text{Hz}}{●}$.
- This selection enables us to enhance DQ!

→ Since we have had 50cm aluminum frames, chose it !!

Copper layout dependence

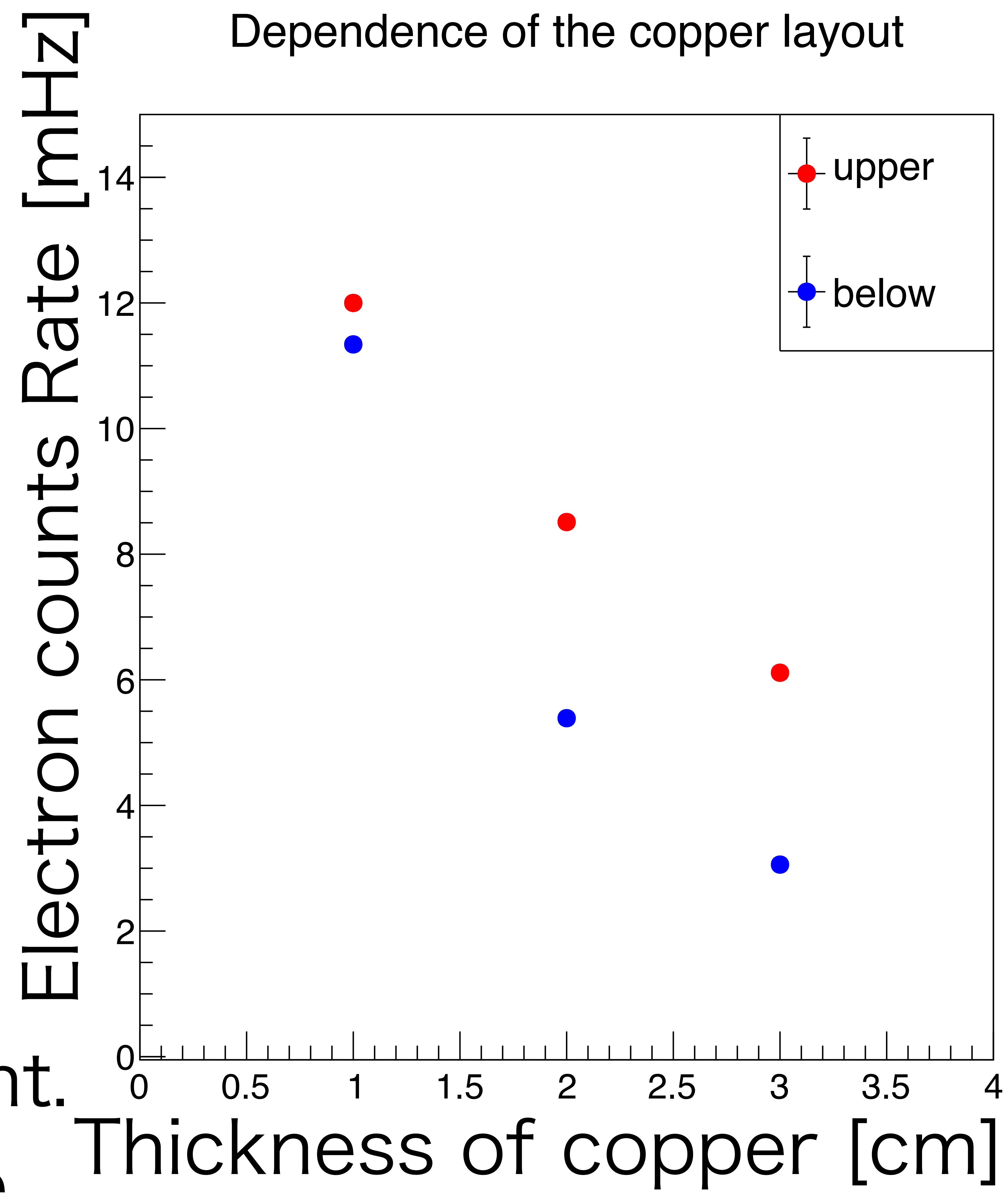
To determine the layout, check the thickness dependence.



	Single	Double	Tripple
Area	◎	○	△
Thickness	△	○	◎

・It's not clear which pattern is the best.

→ Checked using the simulation.



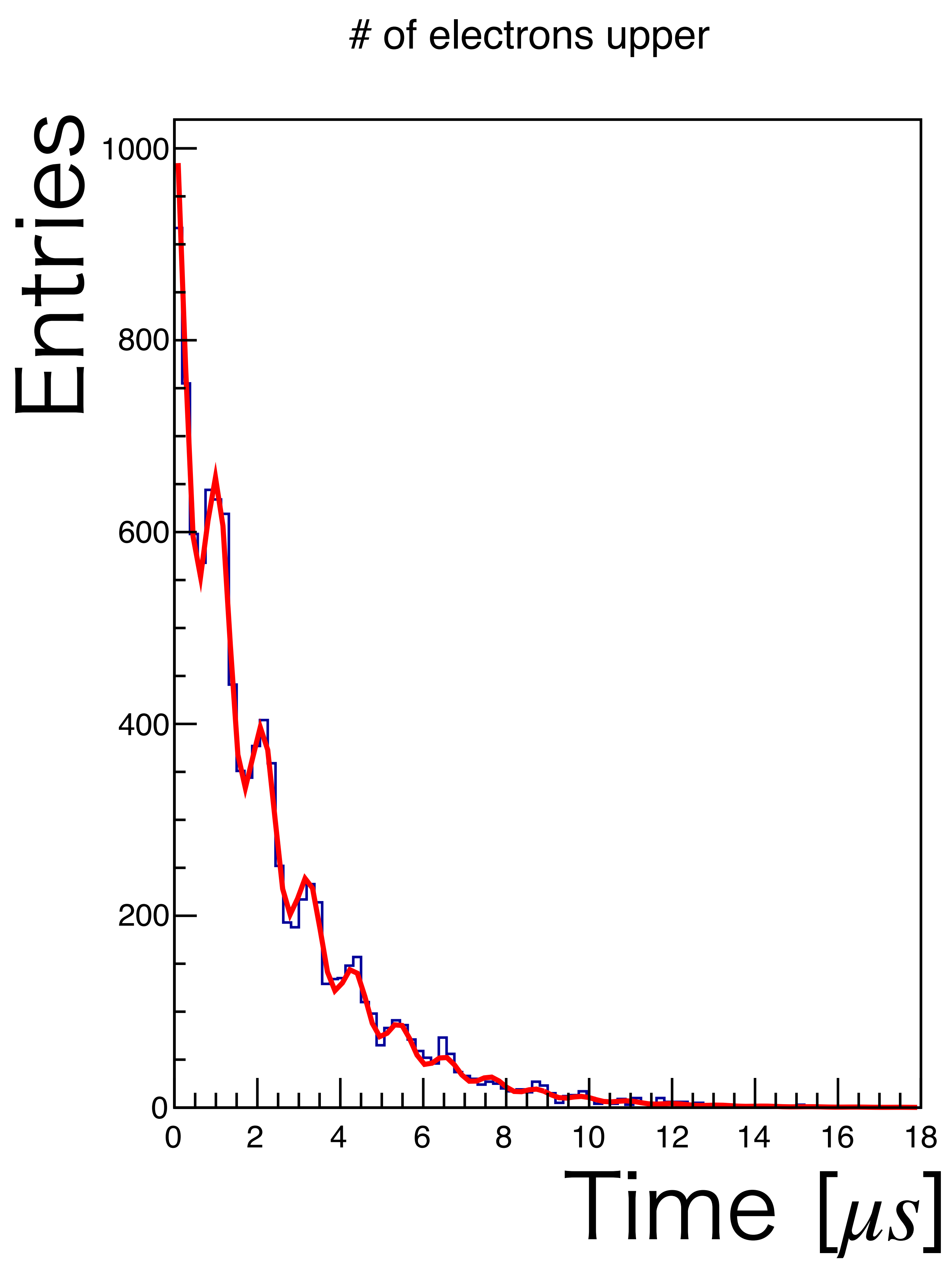
・Right figure shows the size of area is important.

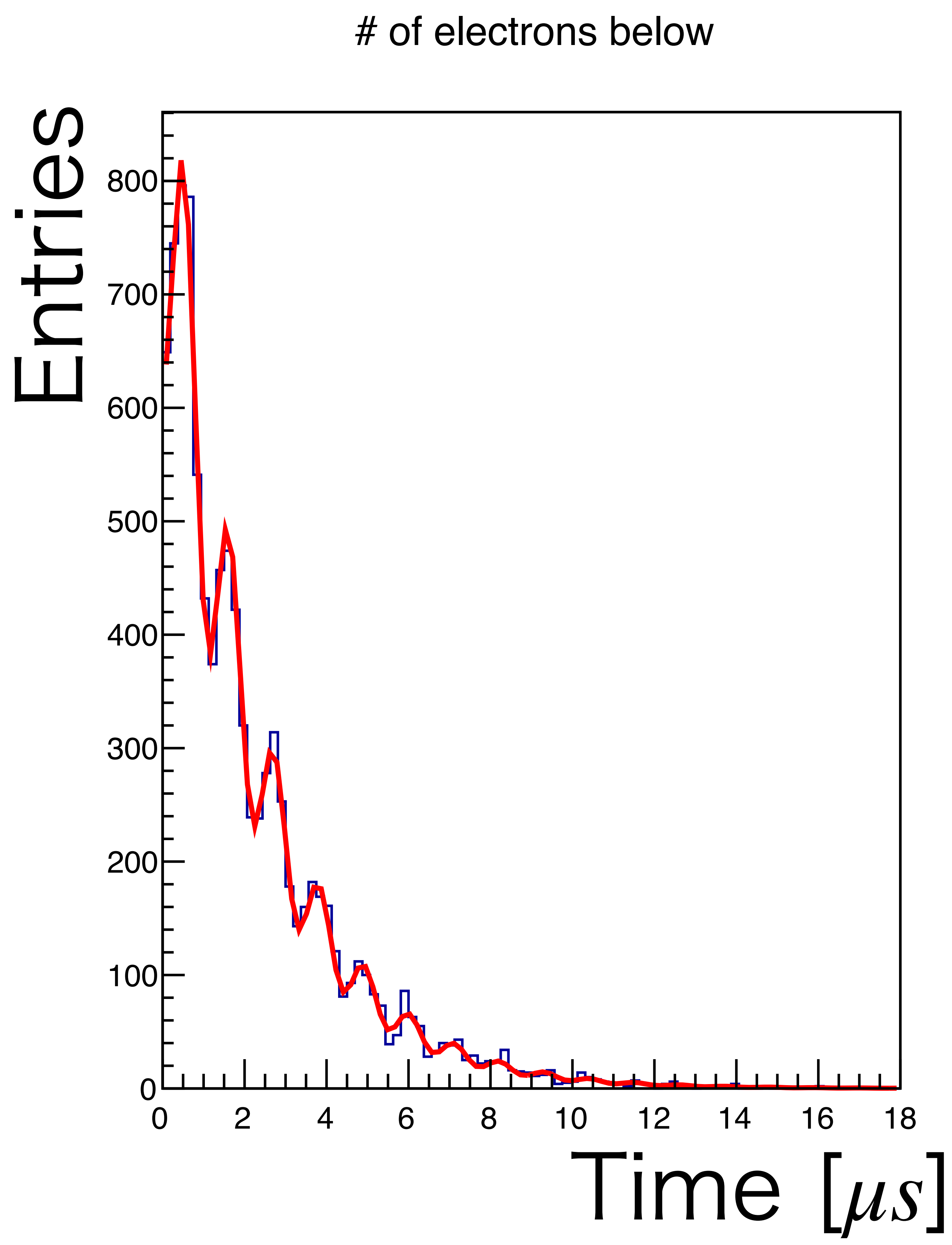
→To maximize # of electrons, chose the left one.

The result of the simulation

・Using the optimized setup, run a simulation.

Condition : Polarization100% $\omega_{set} : 2\pi/\tau_\mu \times 2 = 5.711/\mu s.$





Result

upper: 10,368 # below: 9,797

$\omega : 5.714 \pm 0.016[/\mu s]$

→ Consistent with ω_{set}

ω is calculated within 0.28% error

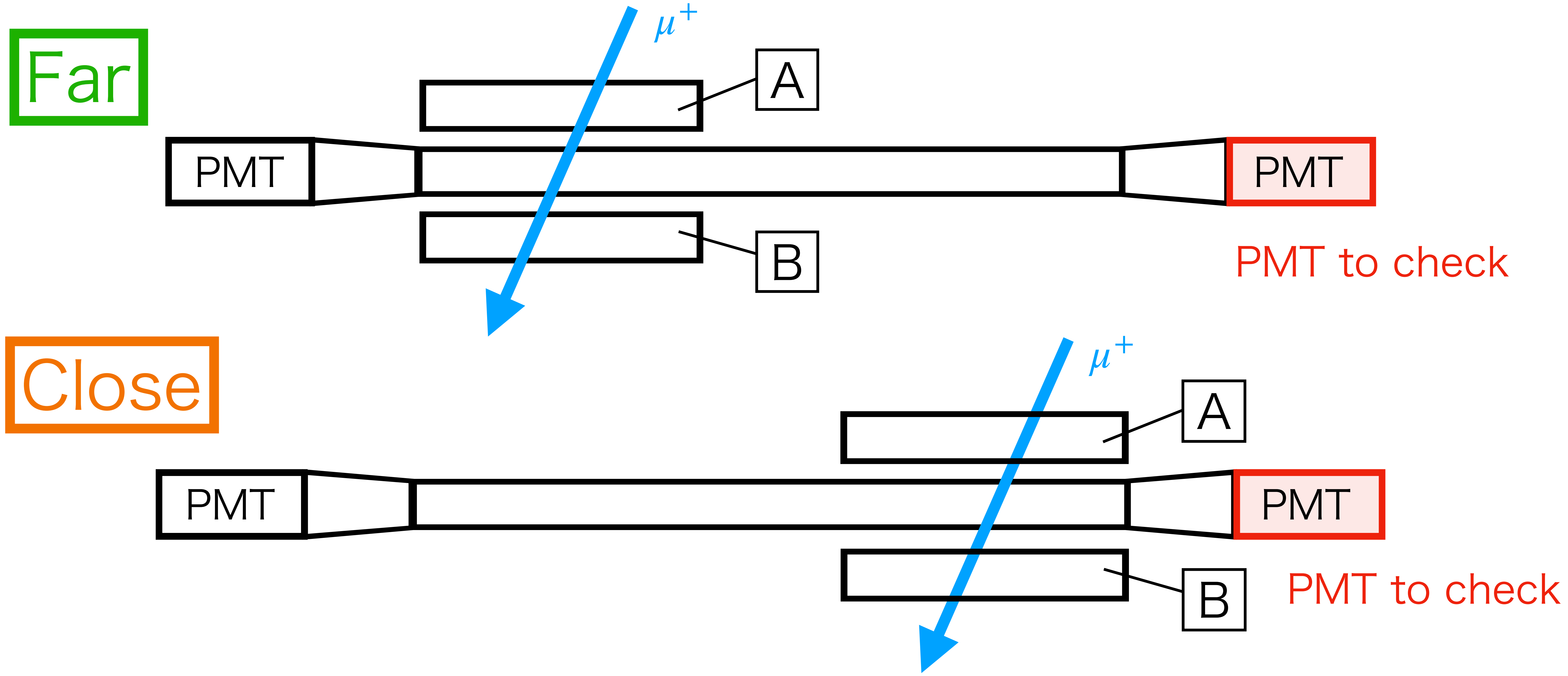
→ Assume B's error =0,

g is determined within 0.28% error

Detector for μ^+ and e^+

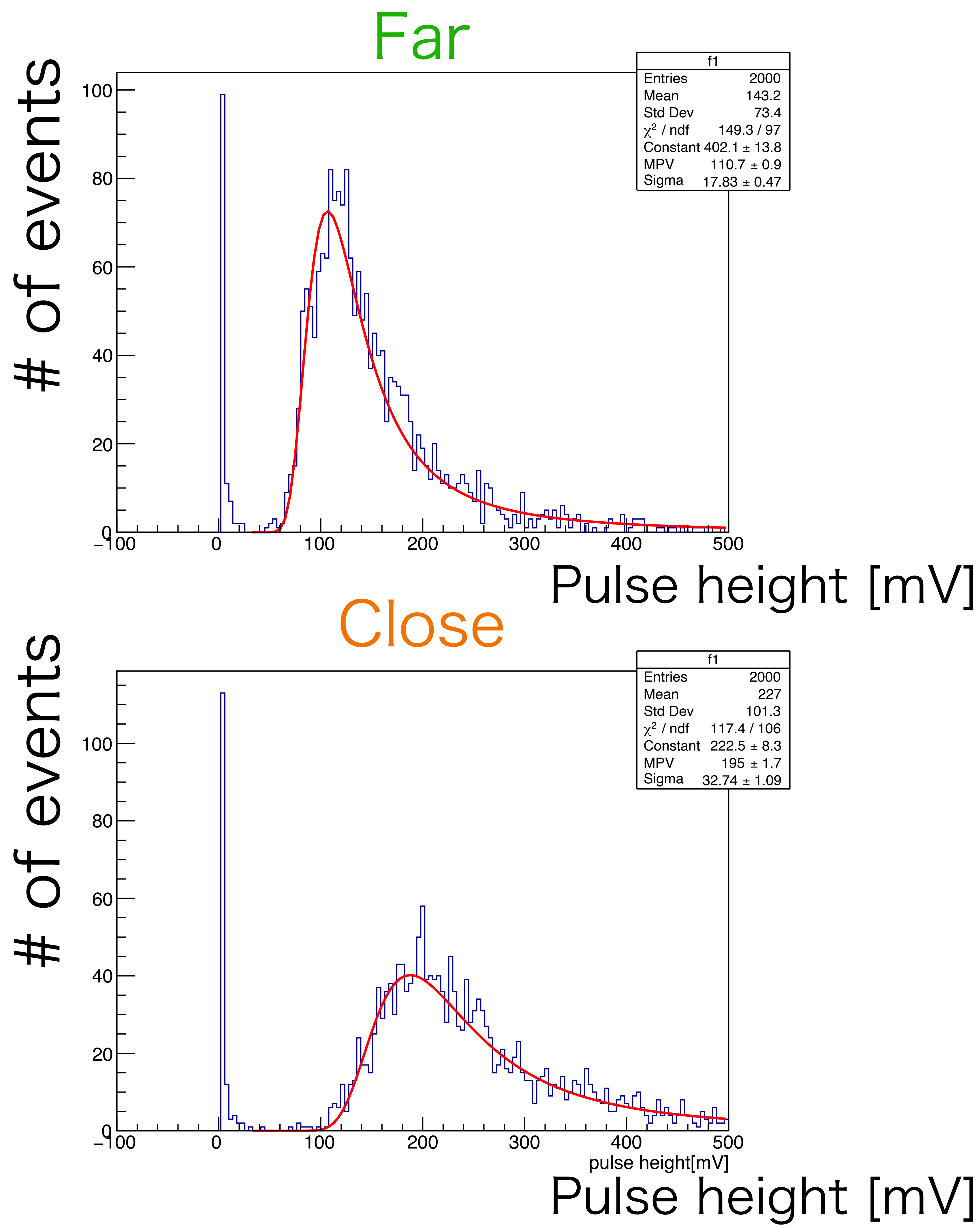
In order to conduct measurement, we constructed detectors for μ^+ and e^+.

→ I measured efficiency of them.



Trigger : A & B coincidence

→ Conduct measurements at both ends for each scintillators.



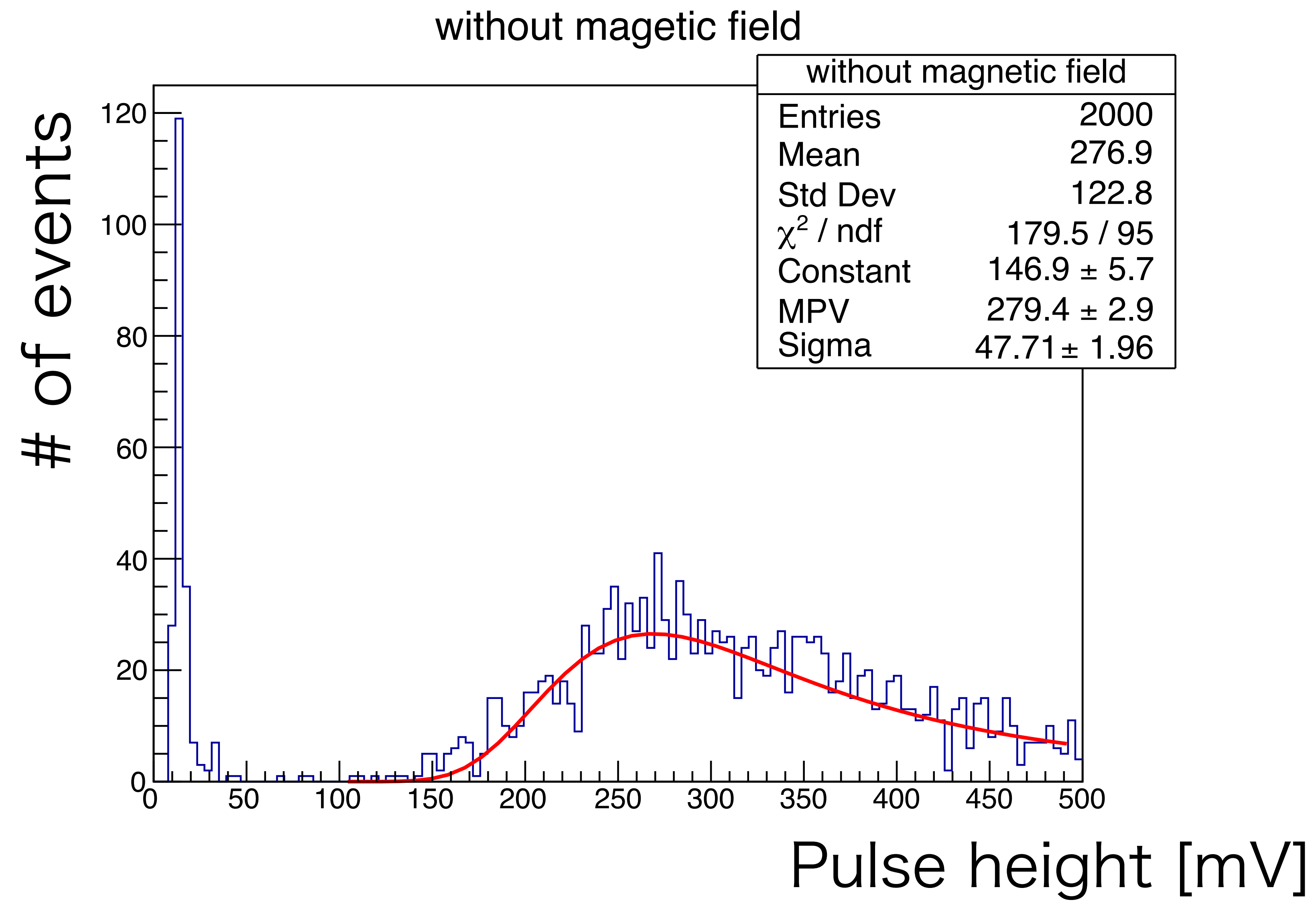
Efficiency & plateau curve

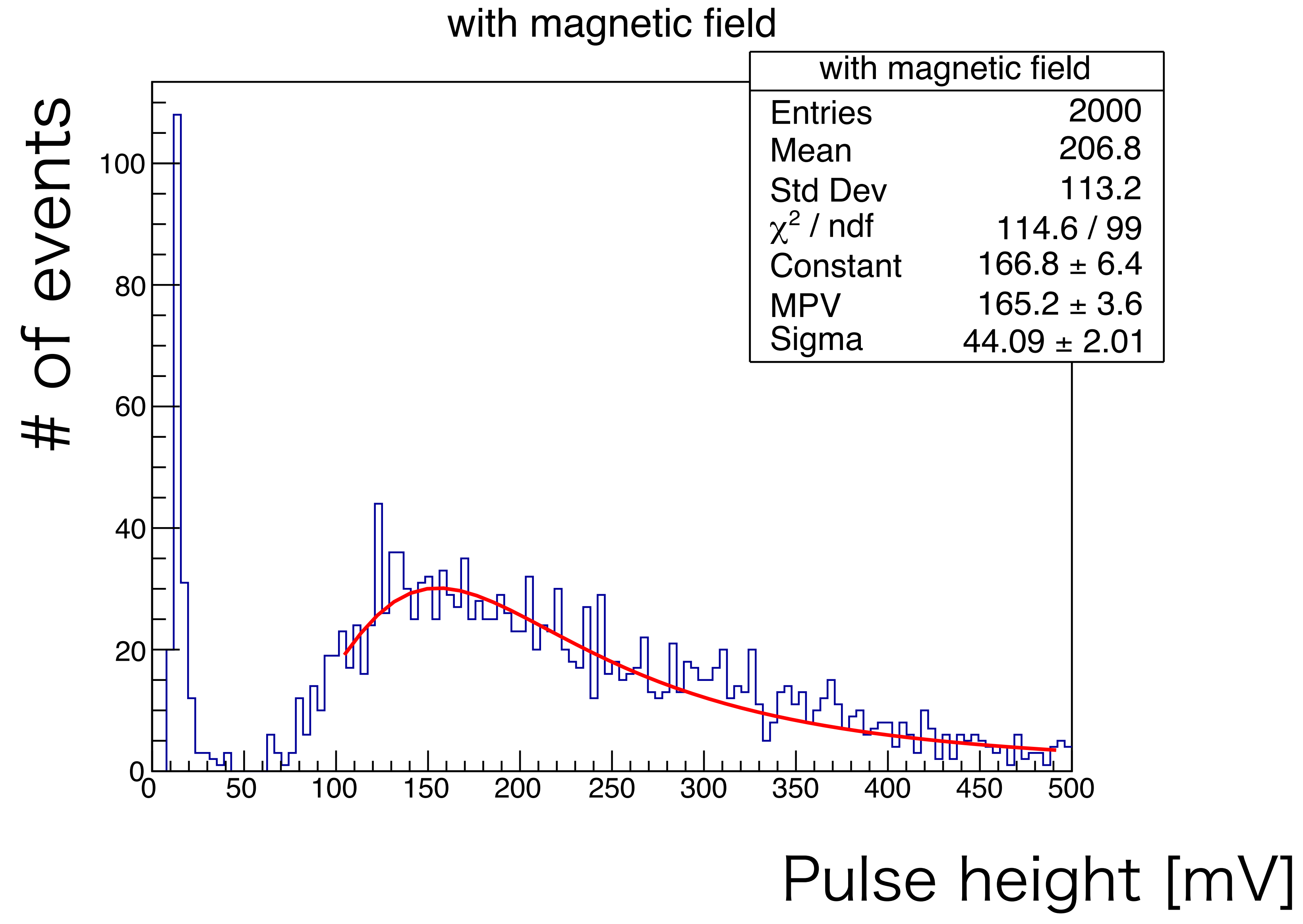
HV values were determined with a margin of ~200V after reaching plateau region.

Without Magnetic field	PMT 0	PMT 1	PMT 2	PMT 3	PMT 4	PMT 5	PMT 6	PMT 7
- HV[V]	2100	1800	2000	2000	1700	2000	2400	2400
Efficiency[%]	95.3±0.5	95.0±0.5	94.0±0.5	93.1±0.6	95.0±0.5	93.9±0.5	91.1±0.6	95.3±0.5

※ efficiencies of PMT 6, 7 are measured at close point.

Effect of magnetic field (gain)





I compared the gain with and without magnetic field.

$$\text{Gain change} = \frac{\text{MPV w/ magnetic field}}{\text{MPV w/o magnetic field}}$$

	PMT 2	PMT 3	PMT 4	PMT 5
Gain change [%]	59.1	99.6	93.9	100

→ the most affected PMT by magnetic field reduced gain by 59.1%.

Effect of magnetic field (efficiency)

To check the effect of magnetic field,

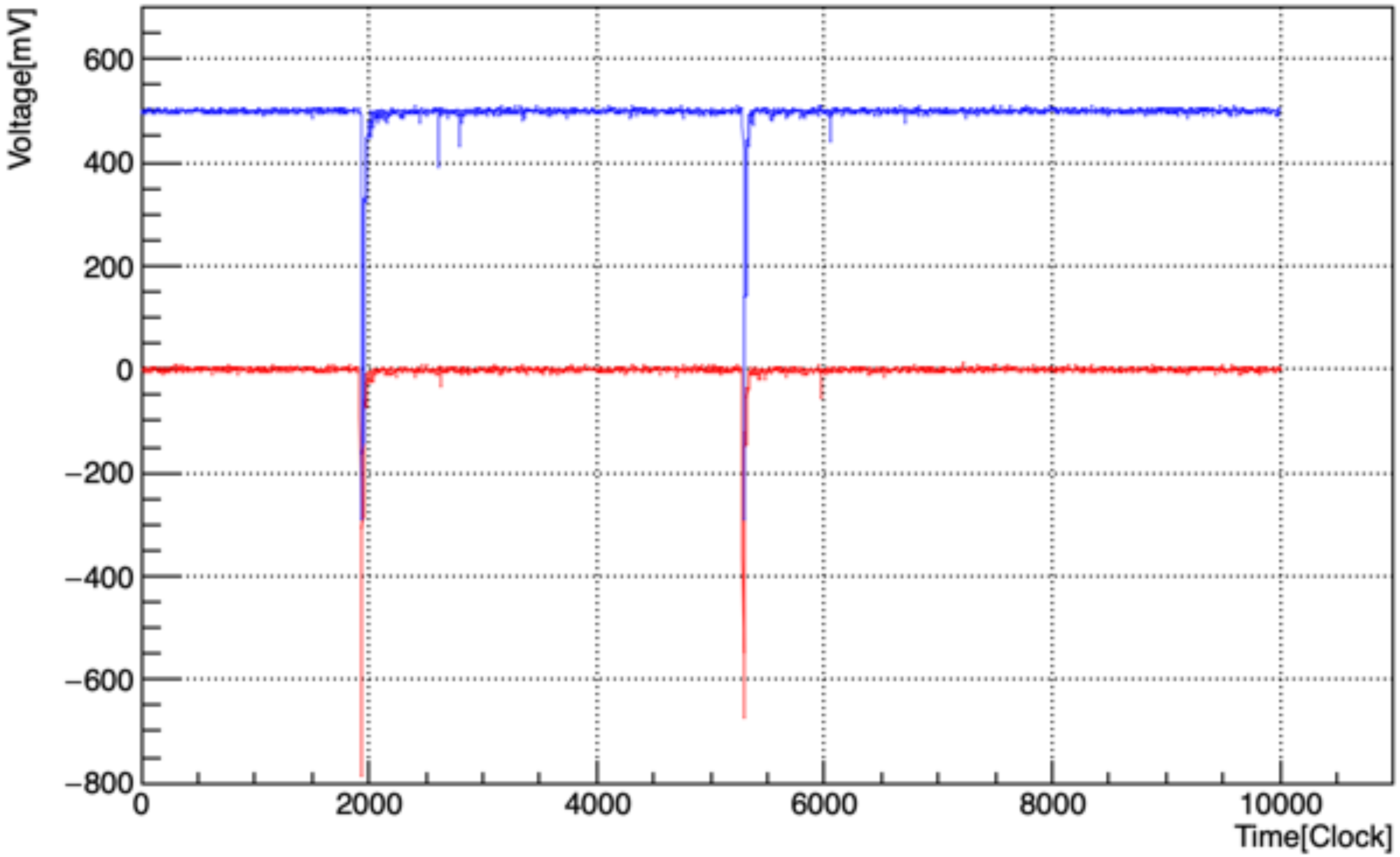
I compared efficiency with and without magnetic field in the same setup

(Use the same HV value)

Efficiency[%]	PMT 2	PMT 3	PMT 4	PMT 5
Without magnetic field	94.1±0.6	93.4±0.6	92.7±0.6	90.4±0.7
With magnetic field	94.3±0.6	93.8±0.6	91.9±0.6	89.7±0.7

→ Efficiencies match within the range of error.

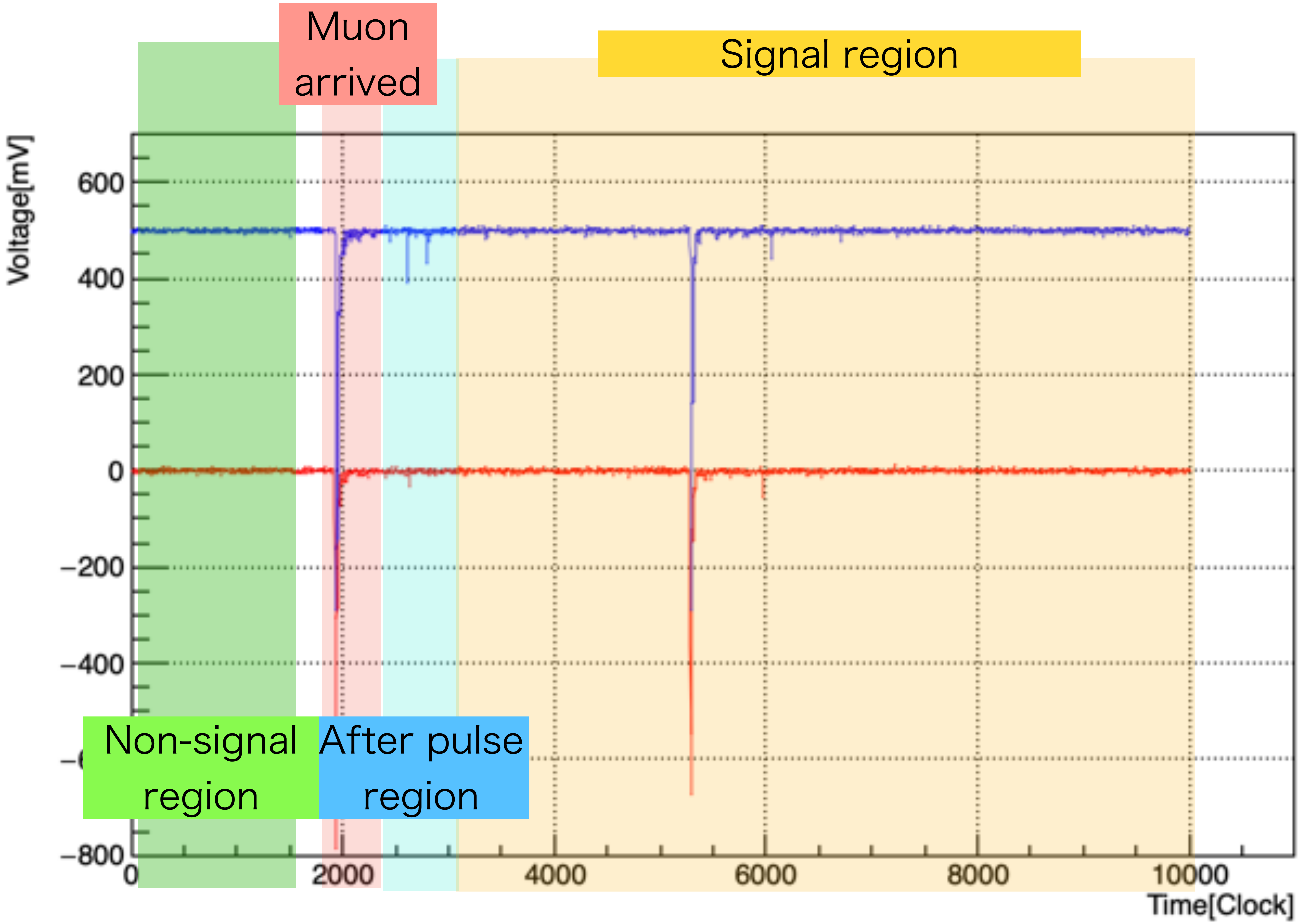
Waveform analysis



- Calculating the pedestal and its std in the non-signal region.
- Over 5σ from the pedestal is considered a pulse.
- Coincidence of pulses in the PMTs at a scintillator

→Signal

Waveform analysis



- Calculating the pedestal and its std in the non-signal region.
- Over 5σ from the pedestal is considered a pulse.
- Coincidence of pulses in the PMTs at a scintillator

→Signal

Event Selection

Events triggered in 6 days(31,000 events)

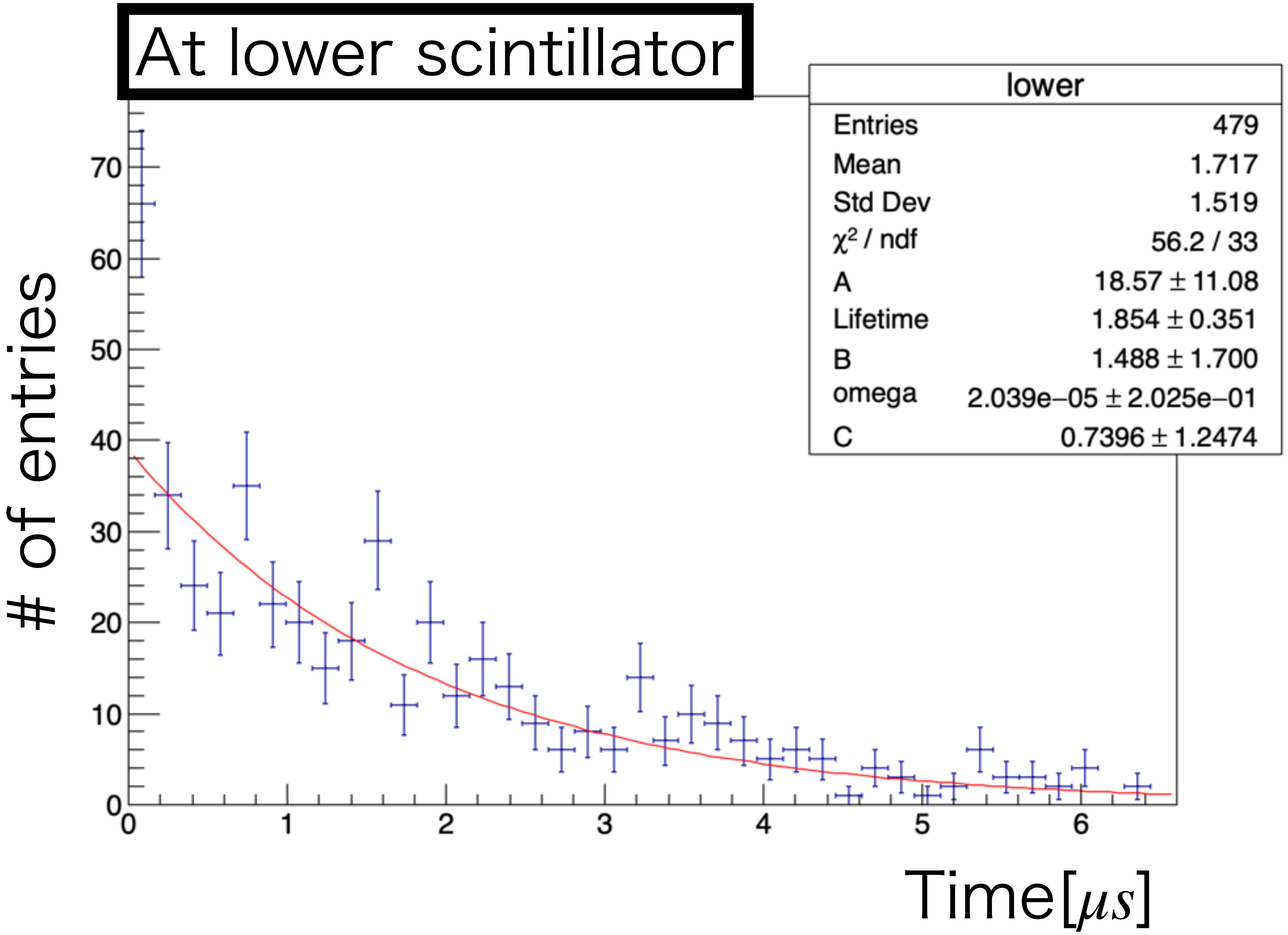
↓

Events cut which has no signal in all PMTs .

↓

Coincidence in both PMTs in a scintillator,
and 60 σ of ground threshold.

Result



Fit the histograms by the function

$$N(t) = A\exp(-t/\tau)(1 + B\cos(\omega t + C))$$

$\rightarrow \tau = 1.85 \pm 0.35[\mu s]$

(Reference 2.20 $[\mu s]$)

Muon decay can be detected.

but ω is zero consistent.

→The g-factor could not measured.

The cause of the deviation

- Simulation yielded 9,797 electrons would be detected below.
  The result was 497.
- What is the cause of this deviation ?
  1. We used 6 days worth of datas. → 60%
  2. Simulation yielded trigger rate is 2.0Hz
     However, the real rate is 1.3Hz →65%
  3. DAQ rate is 0.6Hz → Oscilloscope's Live time is 54%
  4. Set the threshold 60σ.
- By 1~3, we can get only 21% of electrons compared to Simulation.
- By 4, we might lost 1/4 datas.

→ If so, Simulation and the result was consistent.

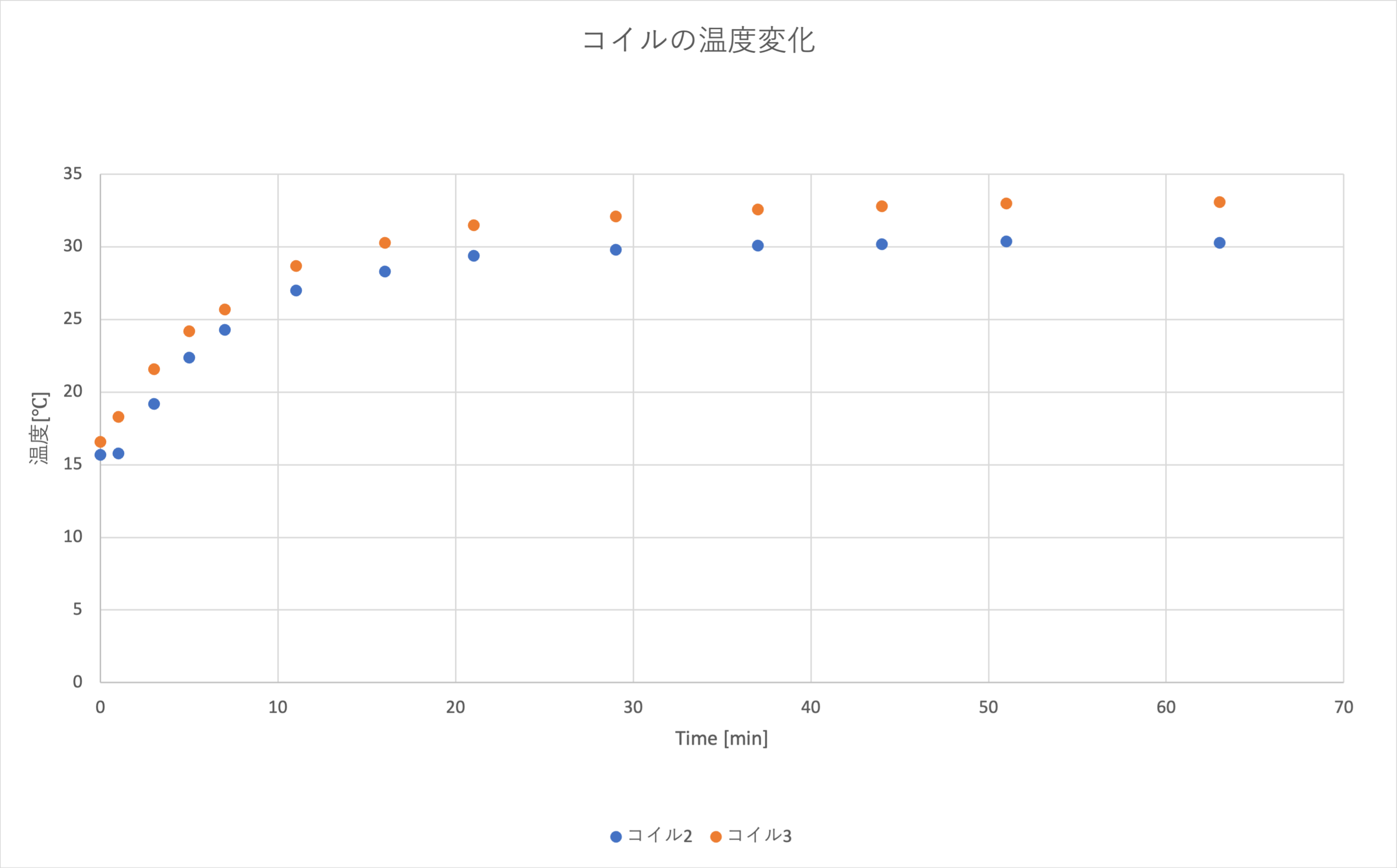
Conclusion

- We made detectors & coils to measure g-factor.
- We were able to measure the life time of muon.
- However, we could't measure ω.

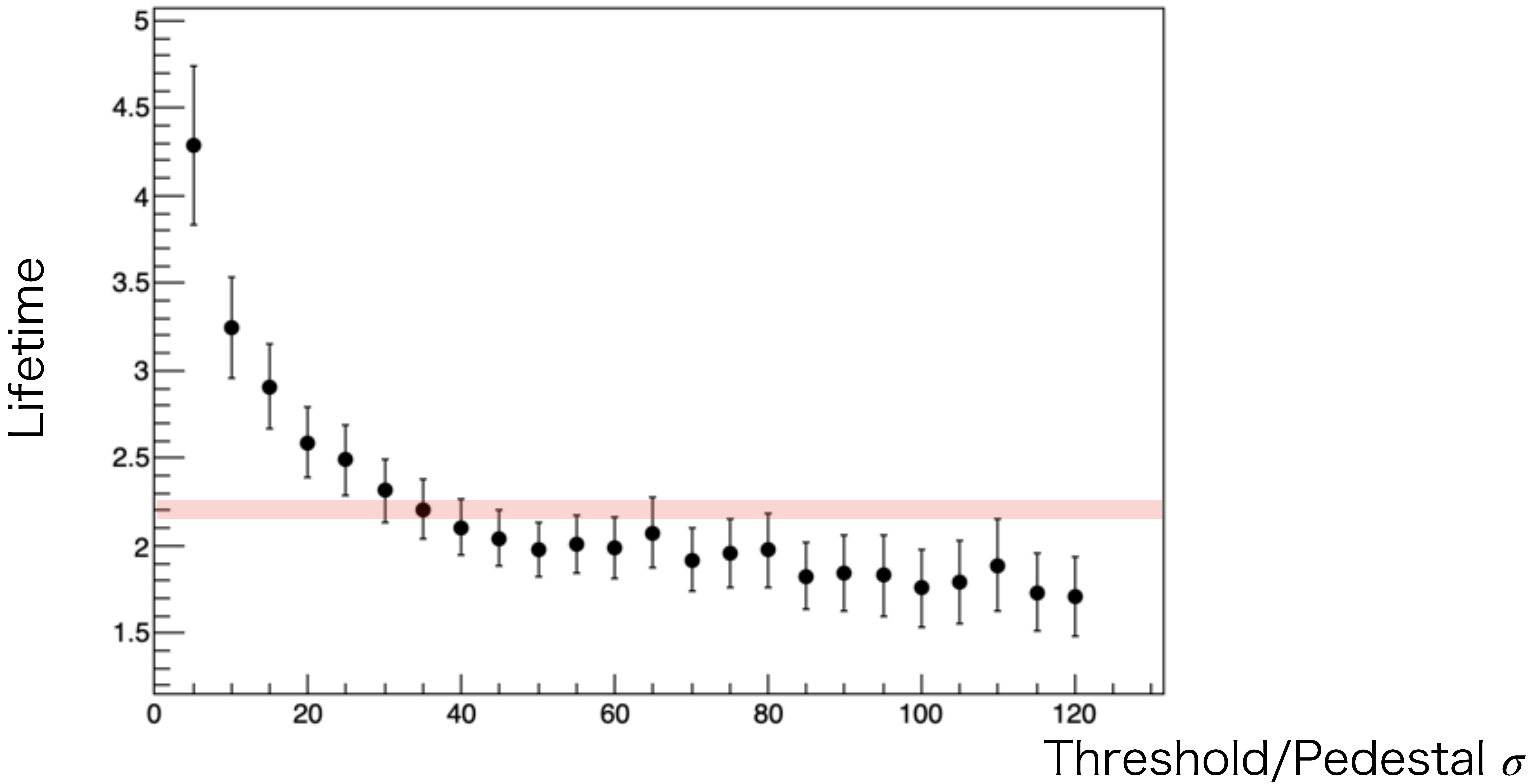
Back-Up

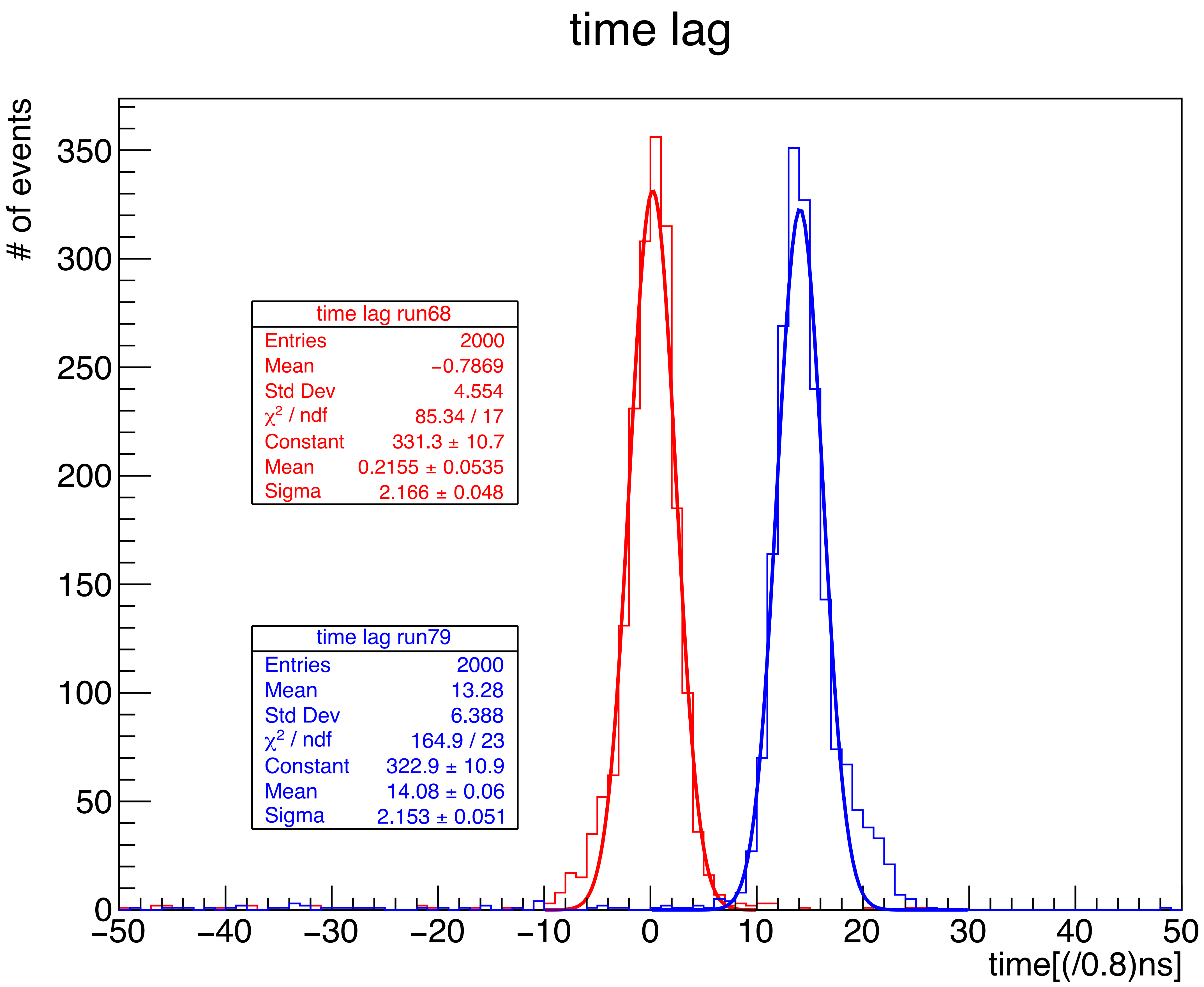
作成の様子

コイルの温度変化



コイルの温度変化
35
30
25
20
15
10
5
0
温度[°C]
0
10
20
30
40
50
60
70
Time [min]
コイル2
コイル3

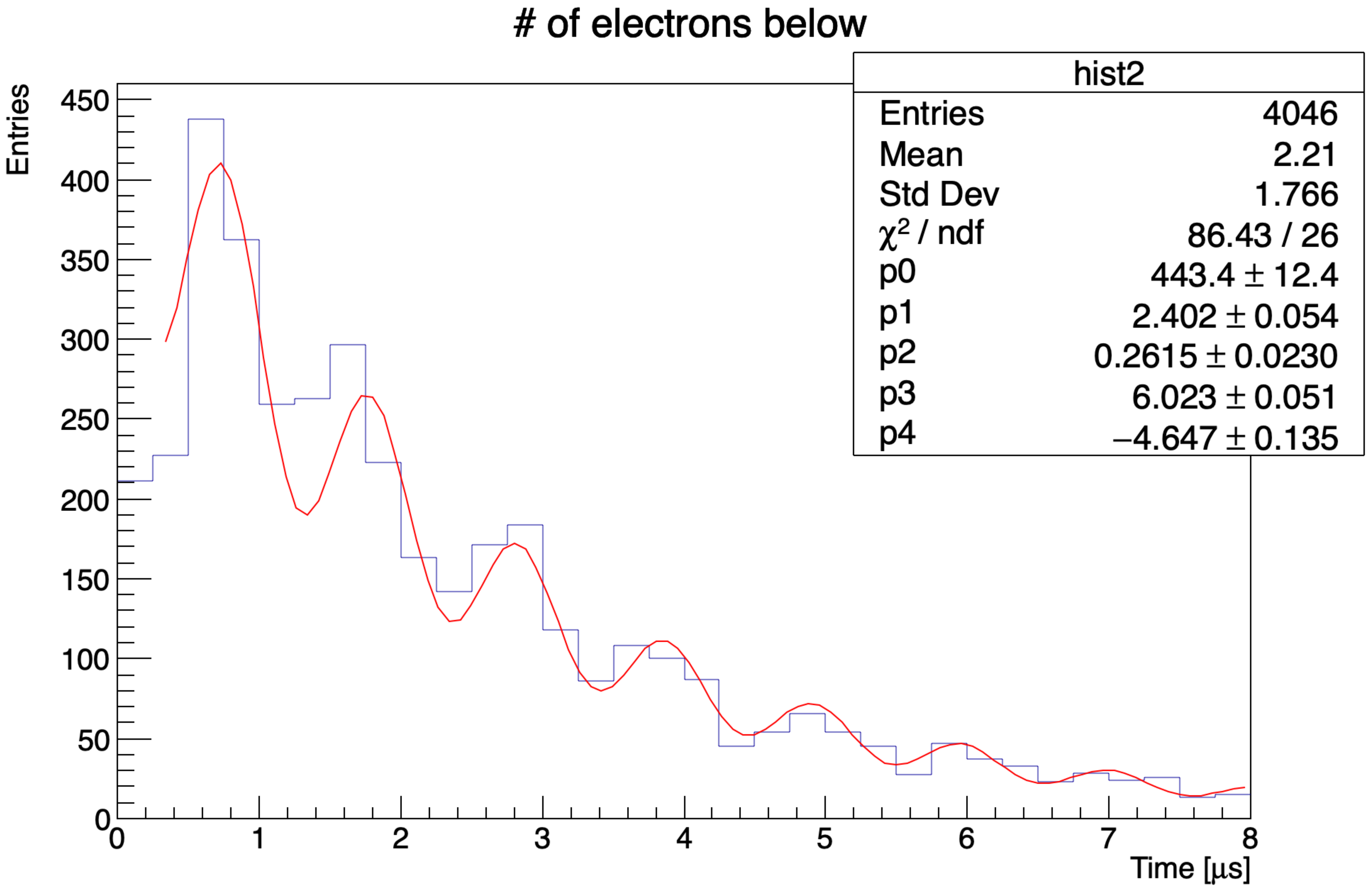

Lifetime
5
4.5
4
3.5
3
2.5
2
1.5
0
20
40
60
80
100
120
Threshold/Pedestal σ



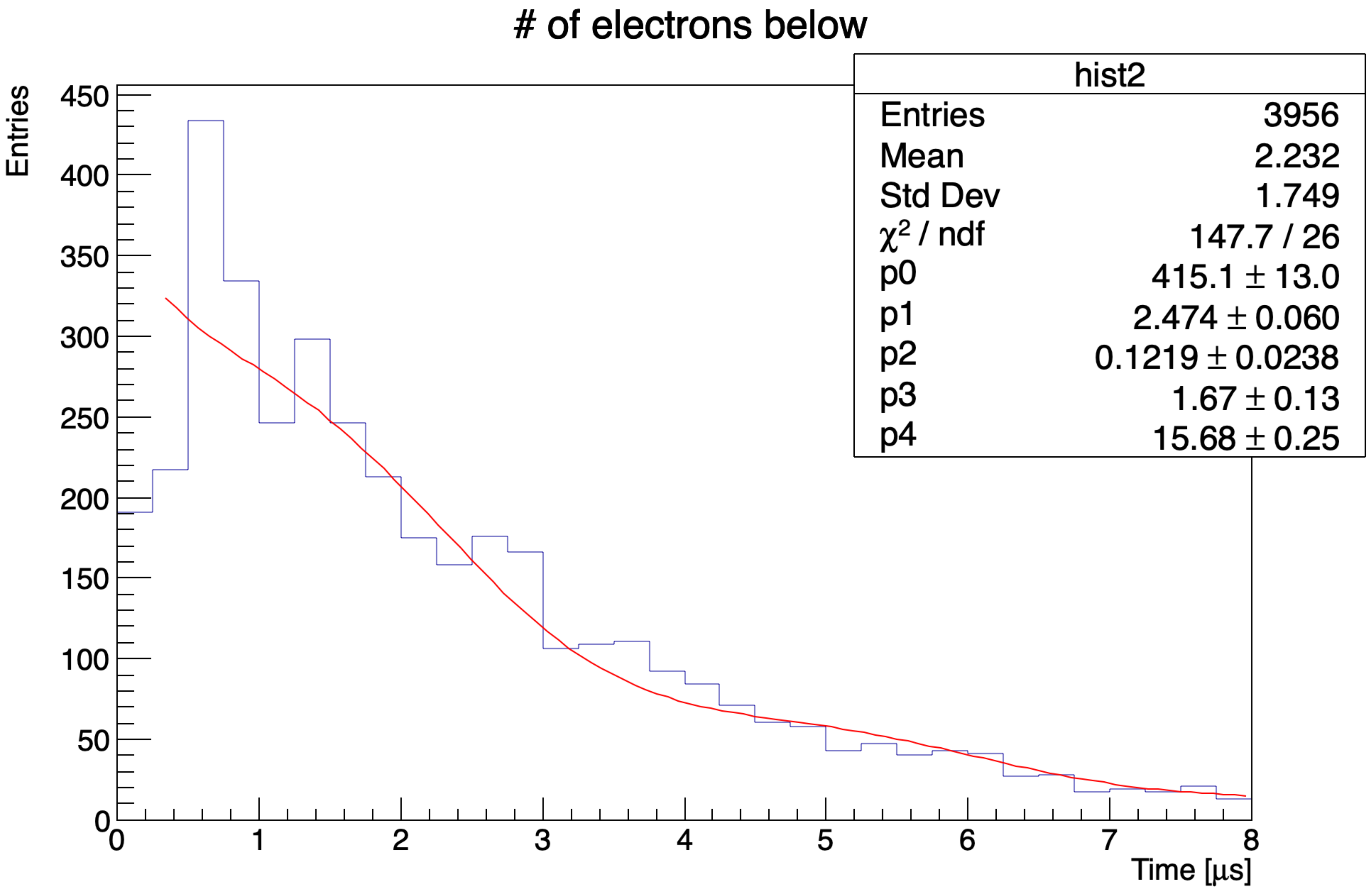
Difference in timing of a signal coming into PMT4 and PMT5

Polarize

90%

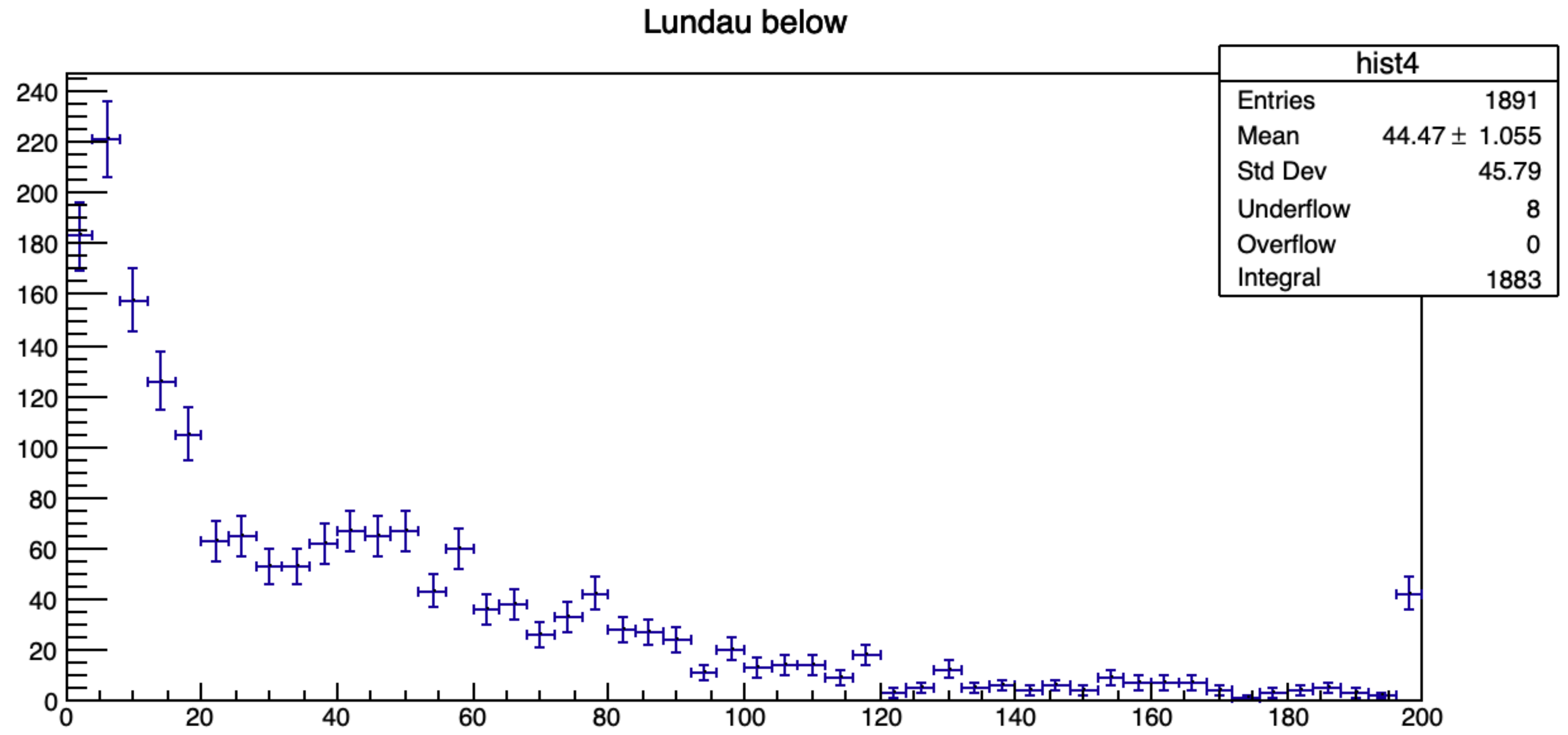


80%

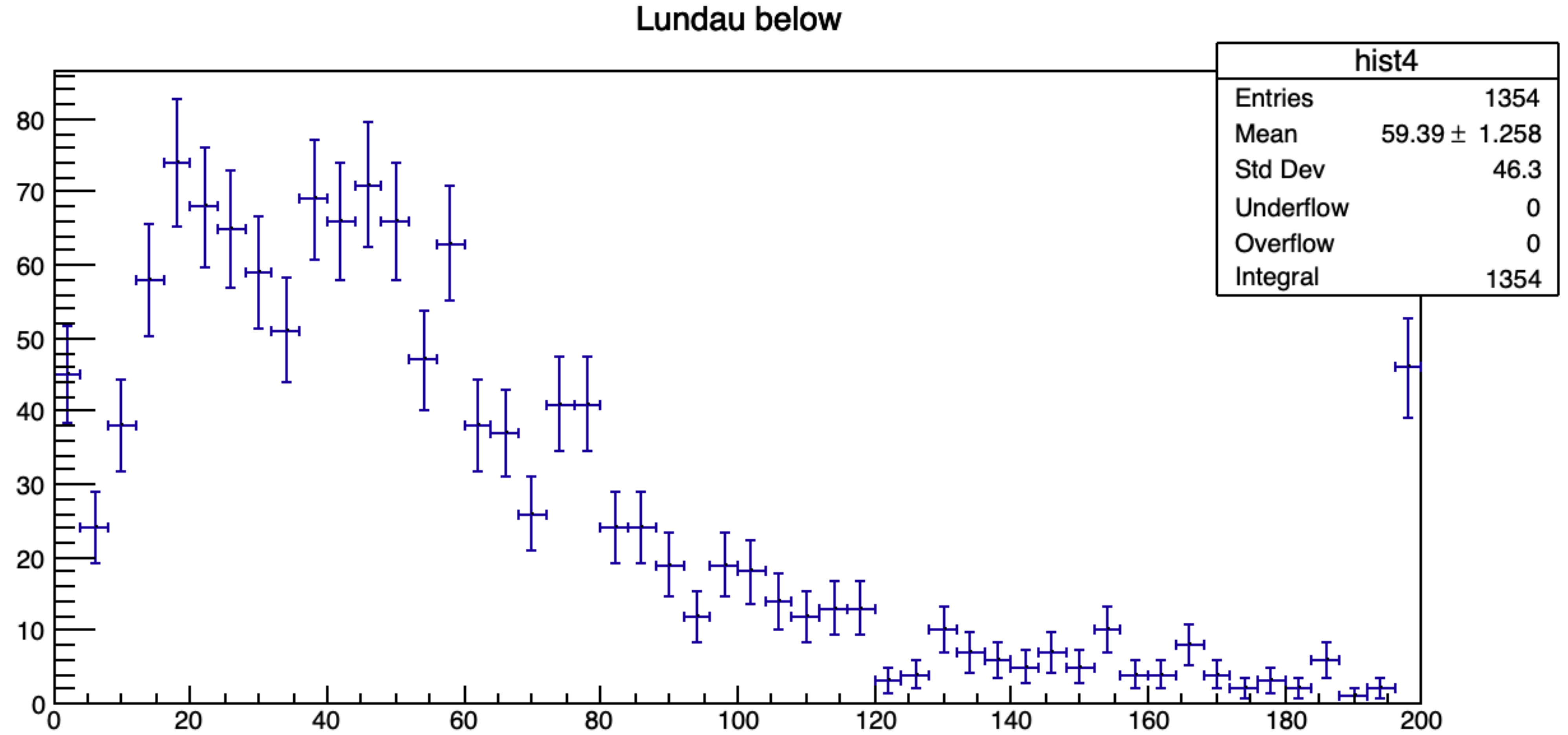


Sigma

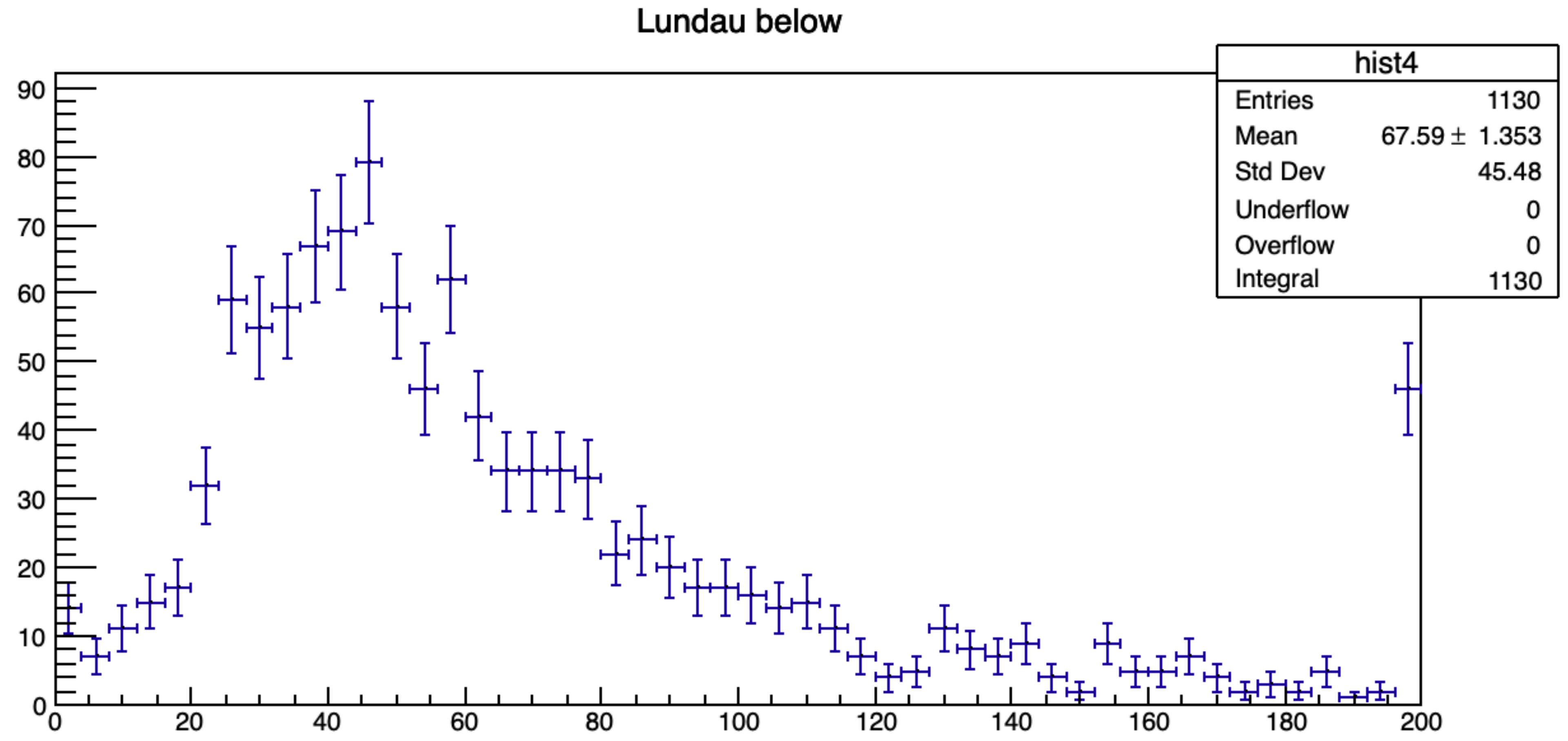
5σ



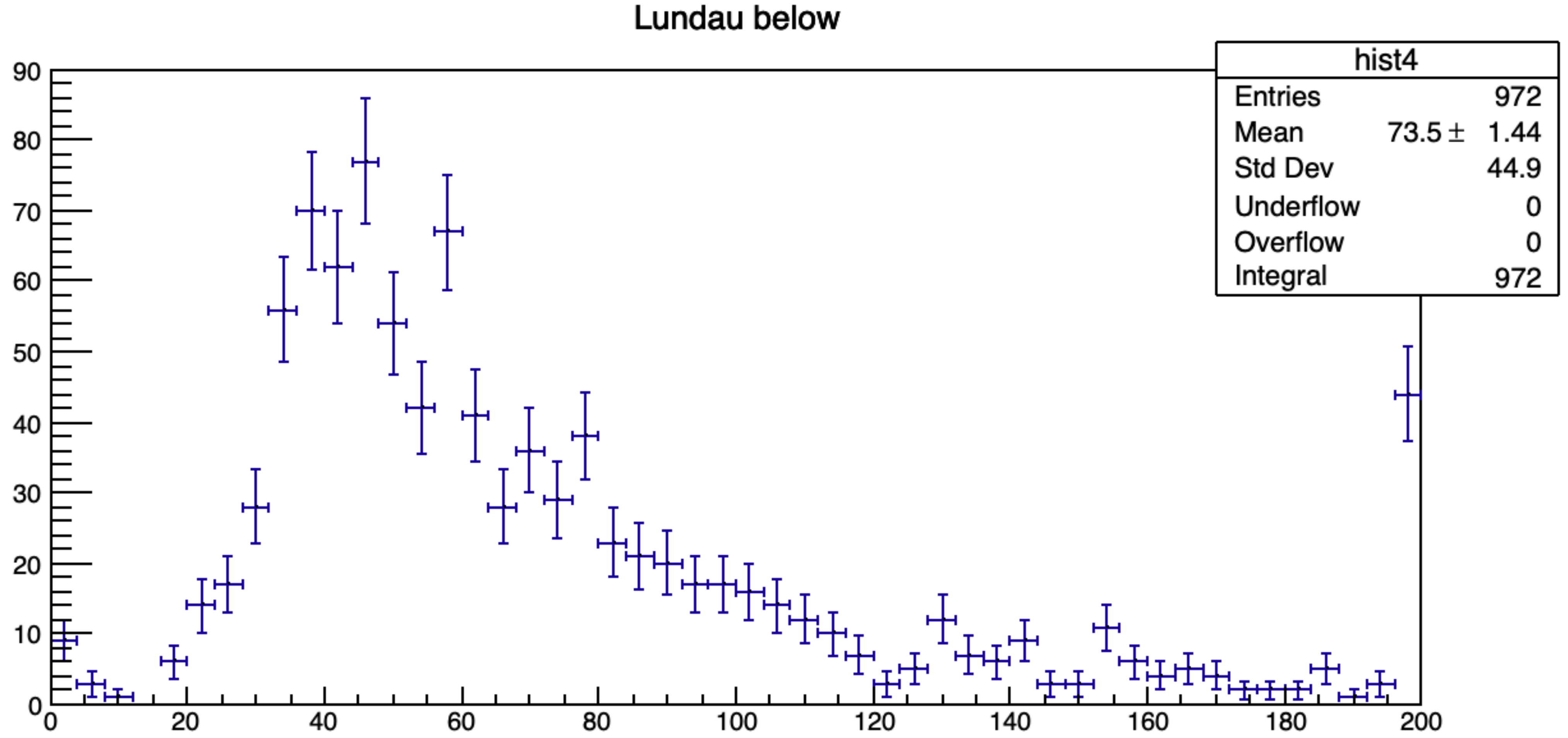
10σ



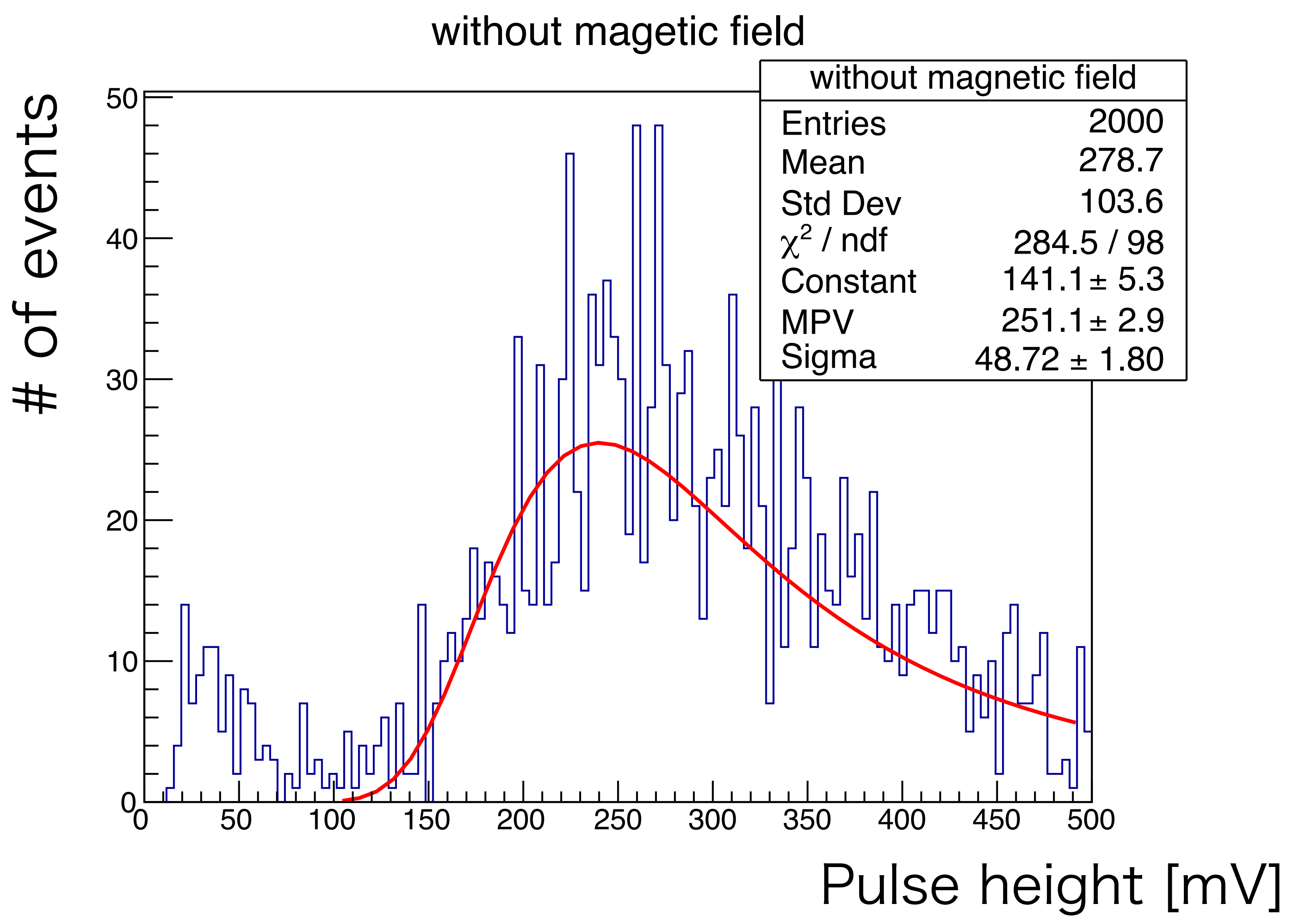
15σ

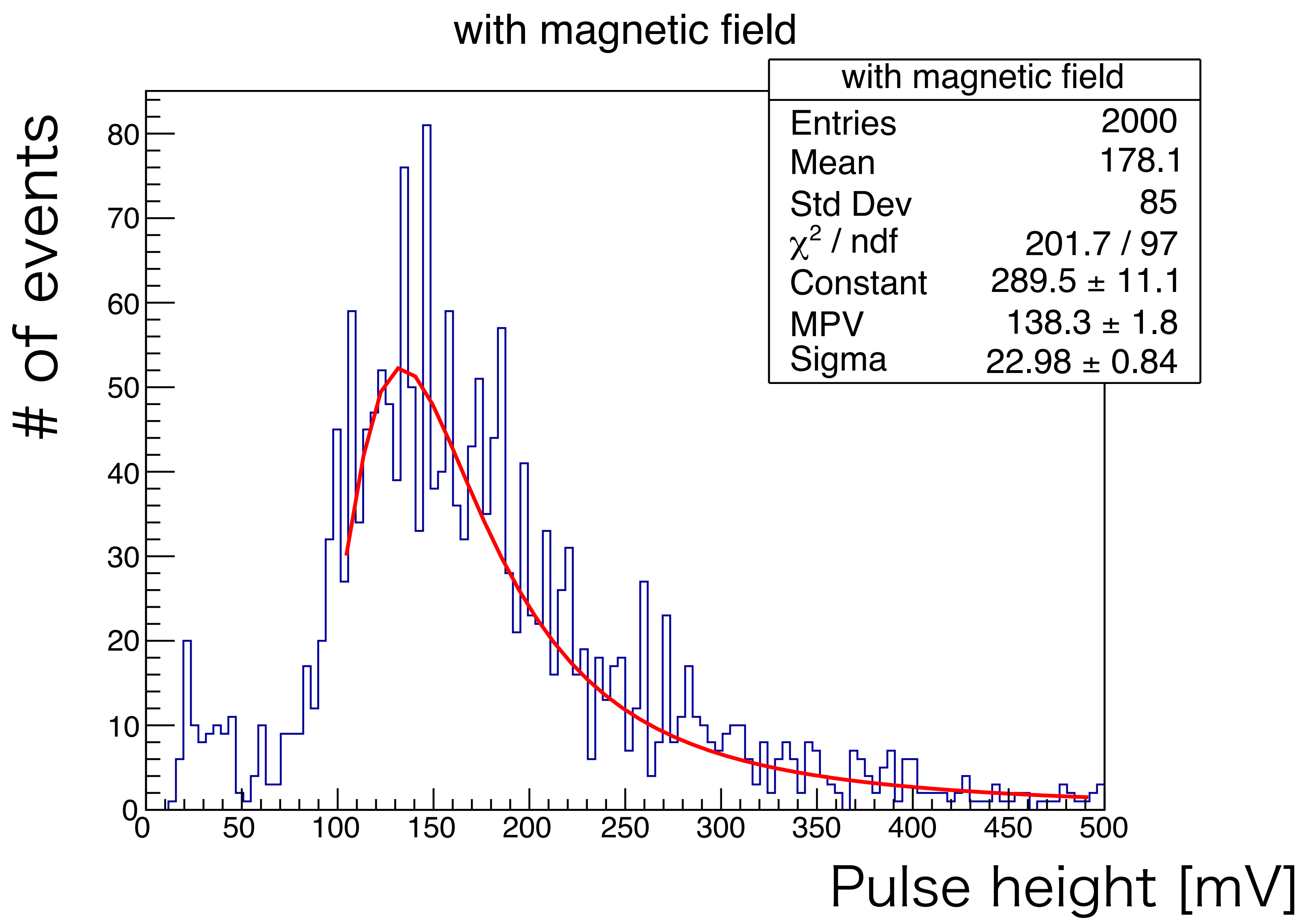


20σ



Effect of magnetic field (gain)





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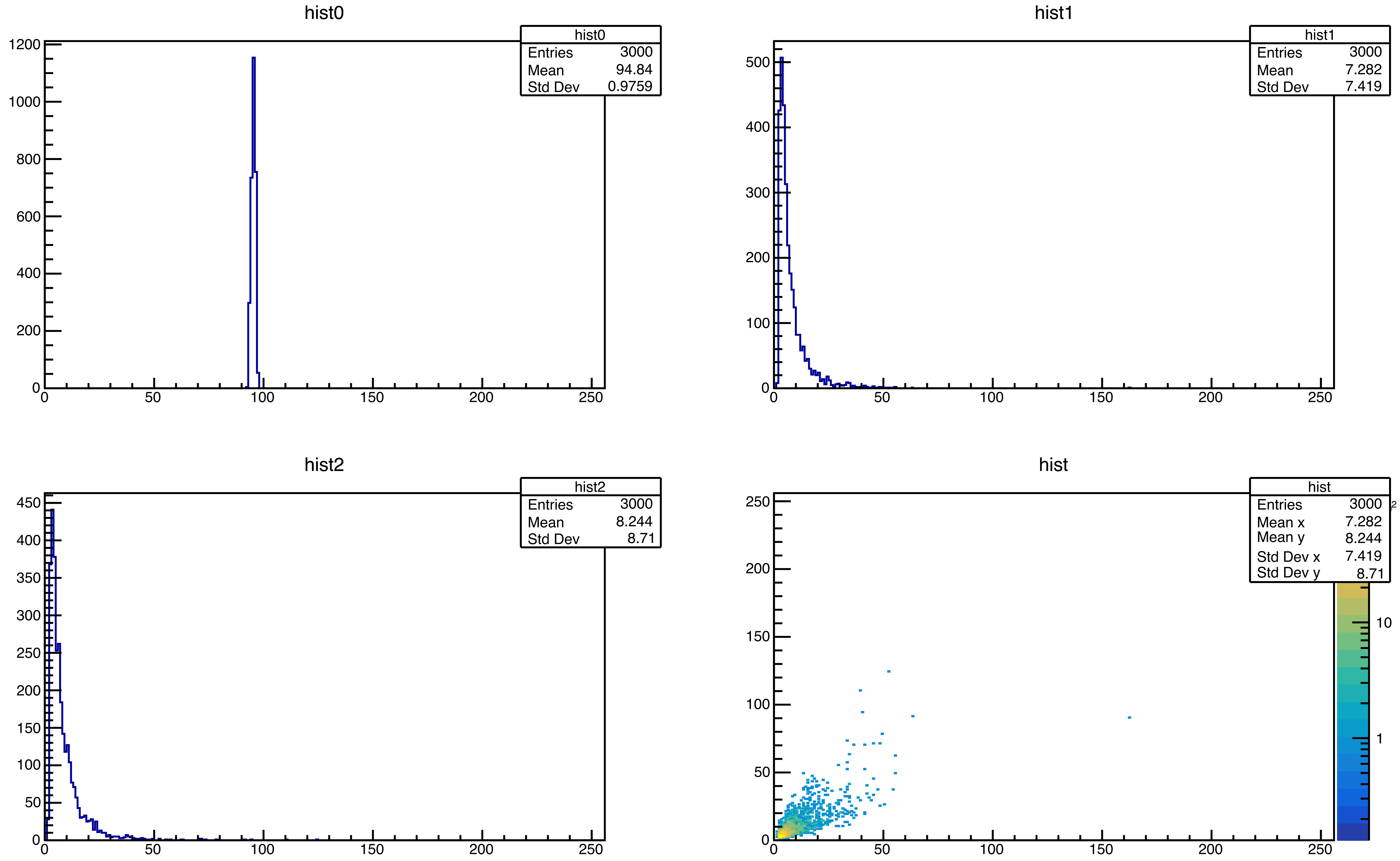
	PMT 0	PMT 1	PMT 2	PMT 3	PMT 4	PMT 5	PMT 6	PMT 7
Gain change [%]	60.0	55.0	59.1	99.6	93.9	100	67.5	65.8

→ the most affected PMT by magnetic field reduced gain by 55%.


hist0
hist0
Entries 3000
Mean 94.84
Std Dev 0.9759
hist1
hist1
Entries 3000
Mean 7.282
Std Dev 7.419
hist2
hist2
Entries 3000
Mean 8.244
Std Dev 8.71
hist
hist
Entries 3000
Mean x 7.282
Mean y 8.244
Std Dev x 7.419
Std Dev y 8.71
0 50 100 150 200 250
10
1

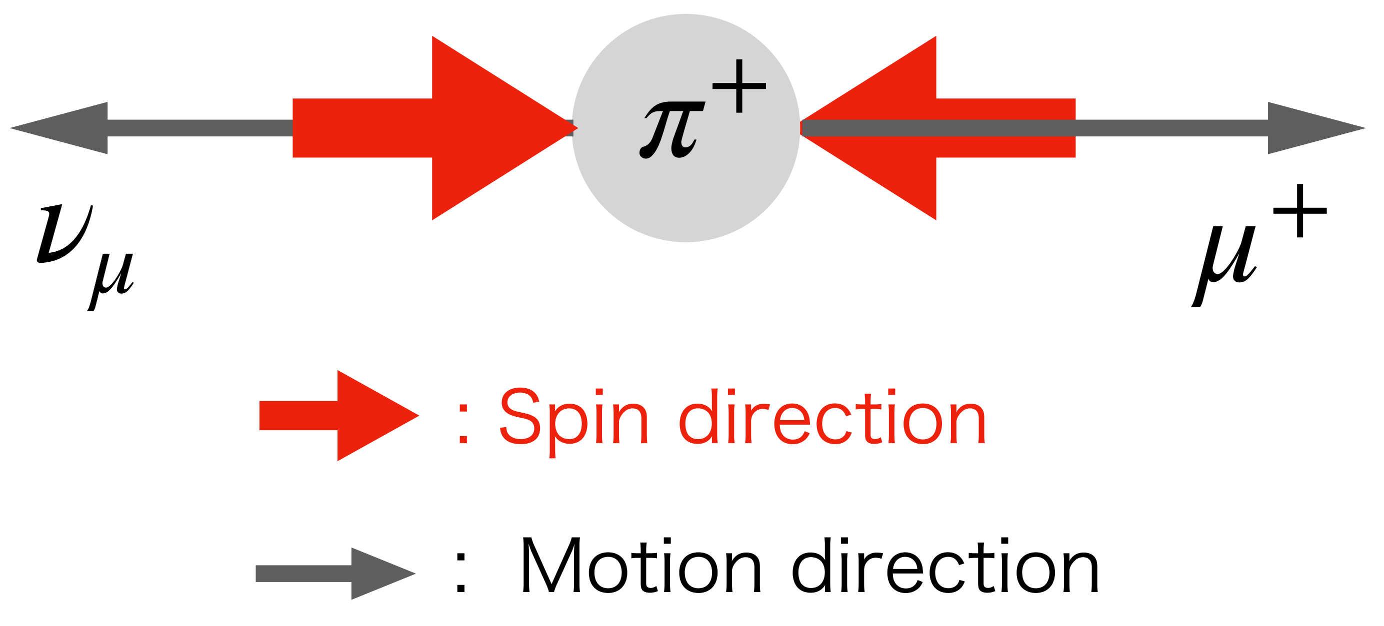
Year-End-Presentation

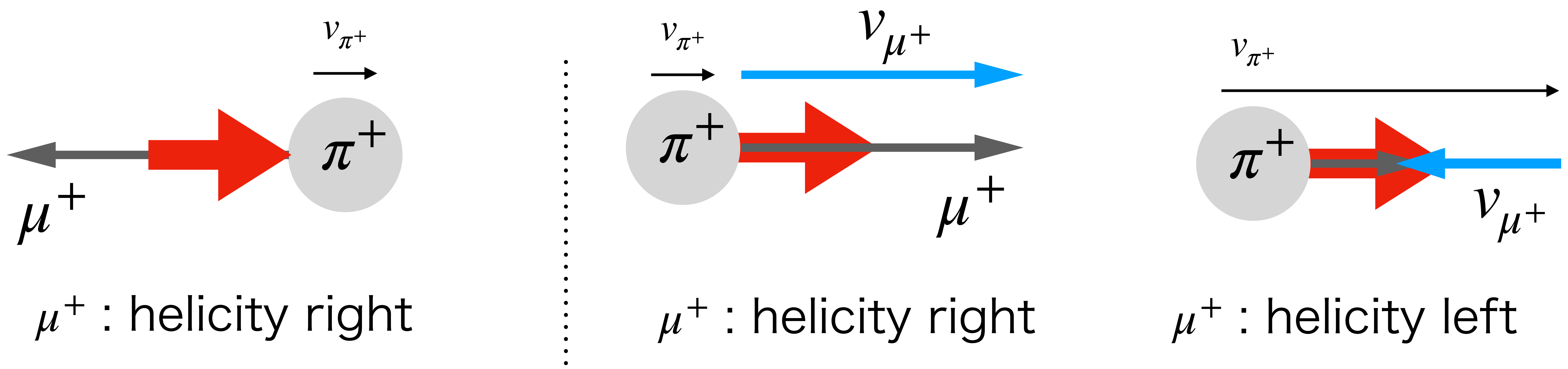
—Optimization for the measurement of the cosmic muon magnetic moment using Simulation

2023/12/14 Sumimura Akinori

Backup

Characteristics of Muon polarization





$\pi^+\mu^+$DIR \ E_{π^+}	**Low**
Same	Helicity Right
Opposite	Helicity Right

Muon Polarized!!!!!

※Spectrum $\pi^+ \propto (E_\pi/M_\pi)^{-\alpha}$

Estimation of available number of muons

- Cosmic muons enter to the LAB after passing through the building

  → Calculate dE/dx

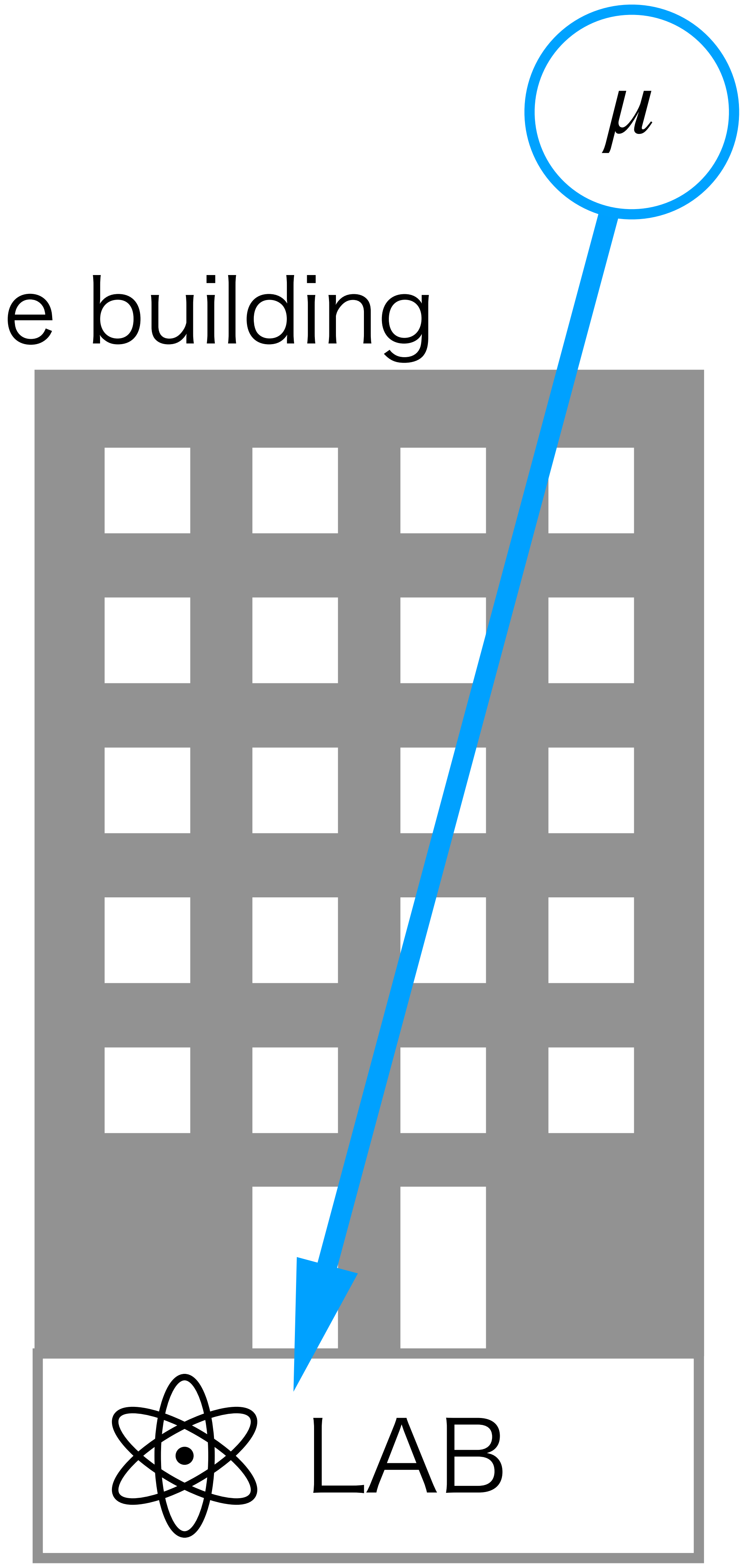
- Let the thickens of concrete $\simeq 30\mathrm{cm}$

  - passing through 8 layers

    → Total path length $\rightarrow 240\mathrm{cm}$

- Let muons dE/dx $\simeq 2\mathrm{MeVcm^2/g}$

  → Mean Energy Loss $\simeq 1.2\mathrm{GeV}$



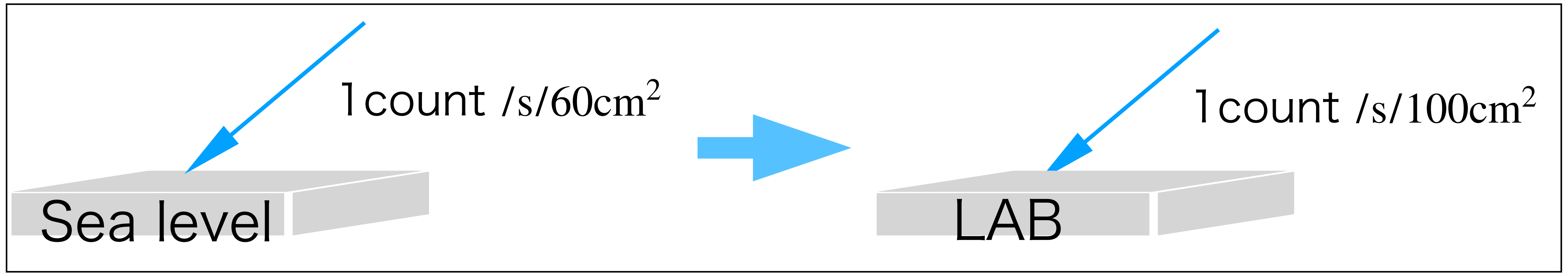
Estimation of available number of muons

- Fig.1 shows the Muon spectrum at sea level
  - Integrate total area derives $0.0087/\mathrm{cm}^2/\mathrm{s}/\mathrm{Sr}$

- As shown, cosmic muons deposit around 1.2GeV

  → muons(momentum ≤ 1.2GeV) are not available

  - $\dfrac{\text{Red Area}}{\text{Total Area}} \simeq 0.4$ → Rate will change like below



→ consistent with Previous Experiment!!

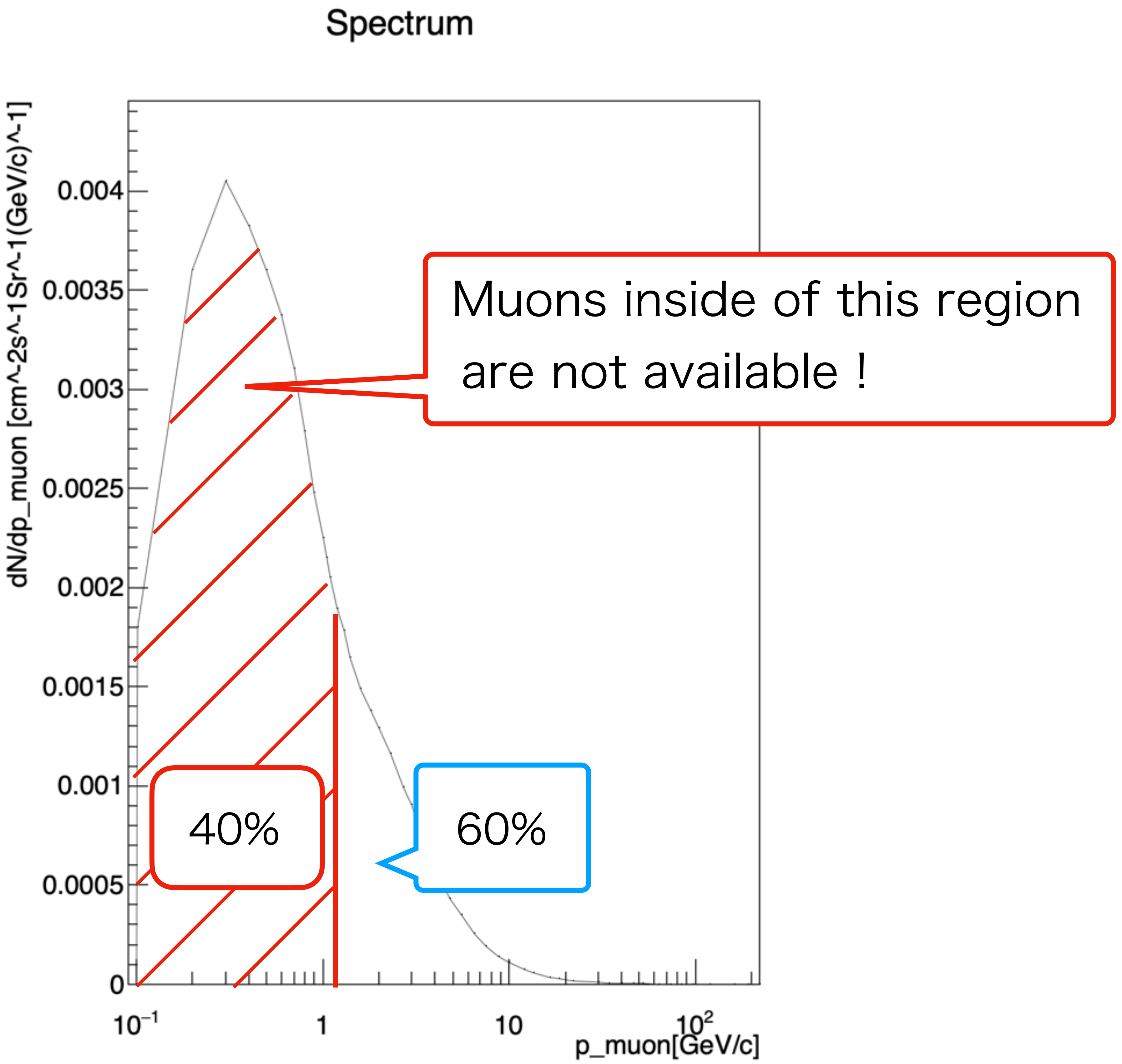
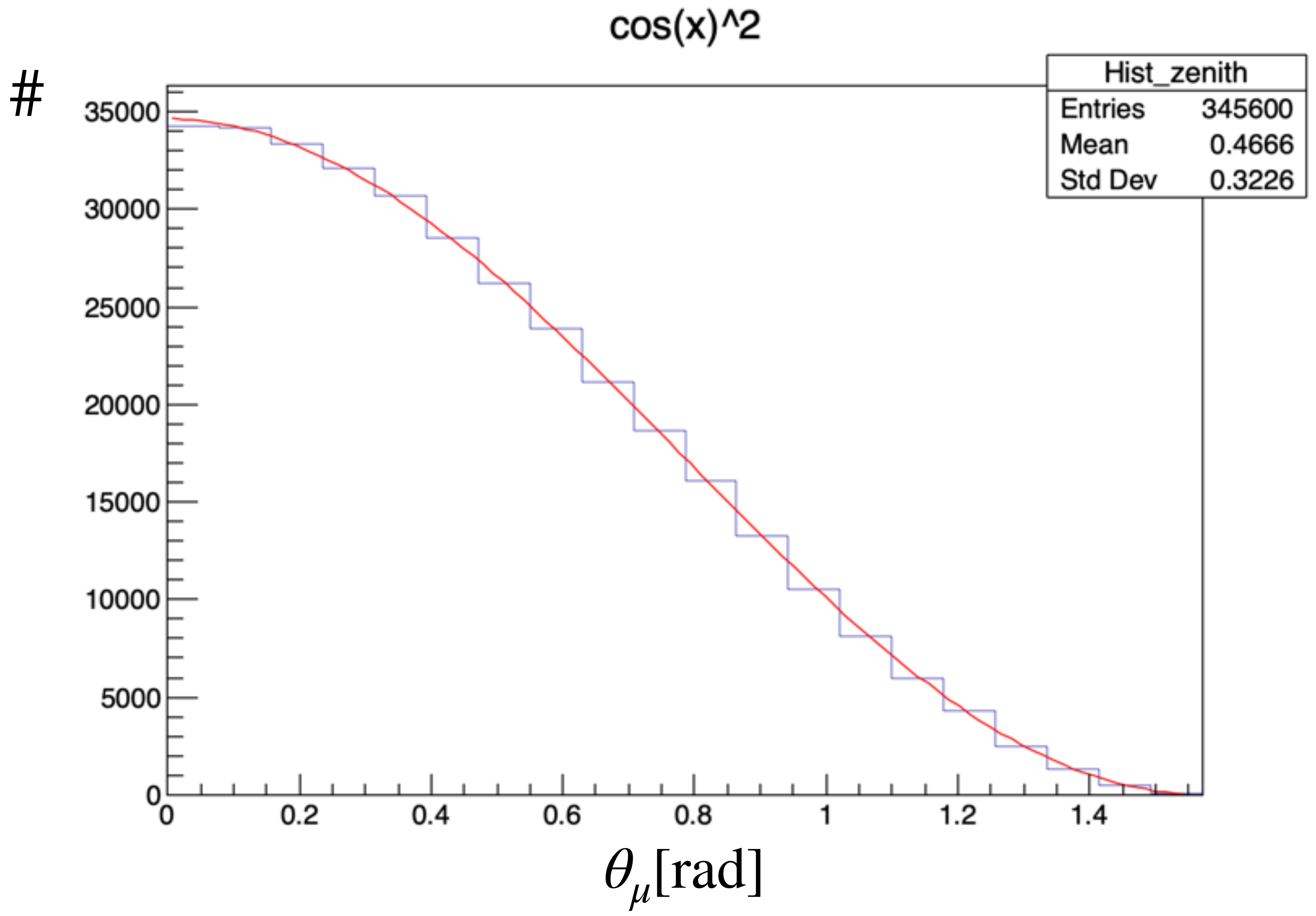


Fig.1 Muon Spectrum at sea level

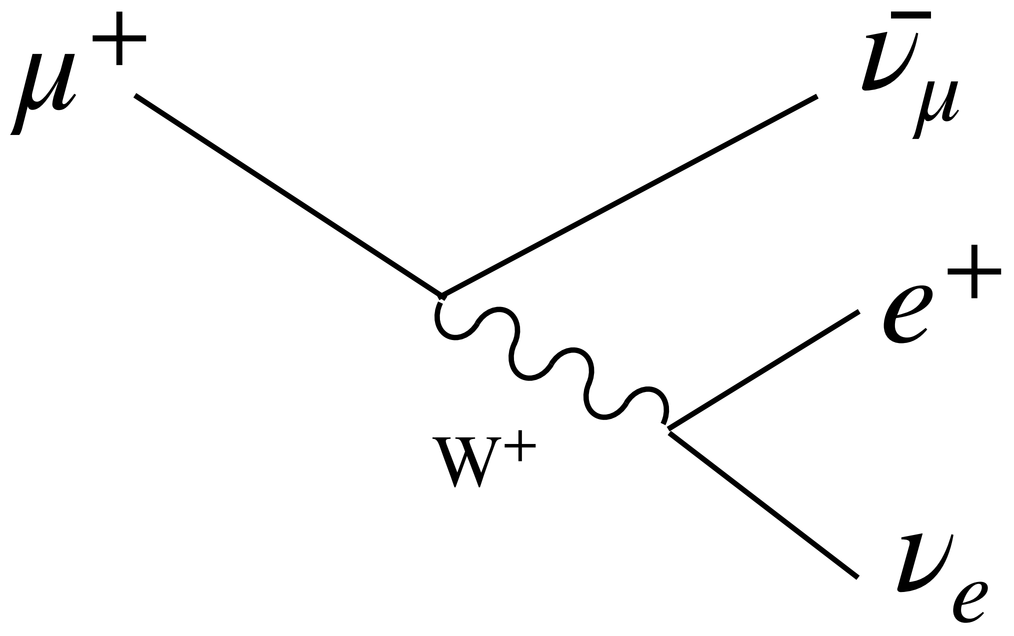
Determination of $E_\mu, \phi_\mu, \mathbf{R}_\mu, \theta_\mu$

- E_μ is determined randomly from the spectrum shown in Fig.1
- ϕ_μ is selected randomly from $[0,2\pi]$
- R_μ is determined randomly $-\text{width} \le x_\mu \le \text{width}$ $-\text{length} \le y_\mu \le \text{length}$
- θ_μ is determined randomly from the distribution $cos^2\theta$



Determination of E_e, ϕ_e, θ_e

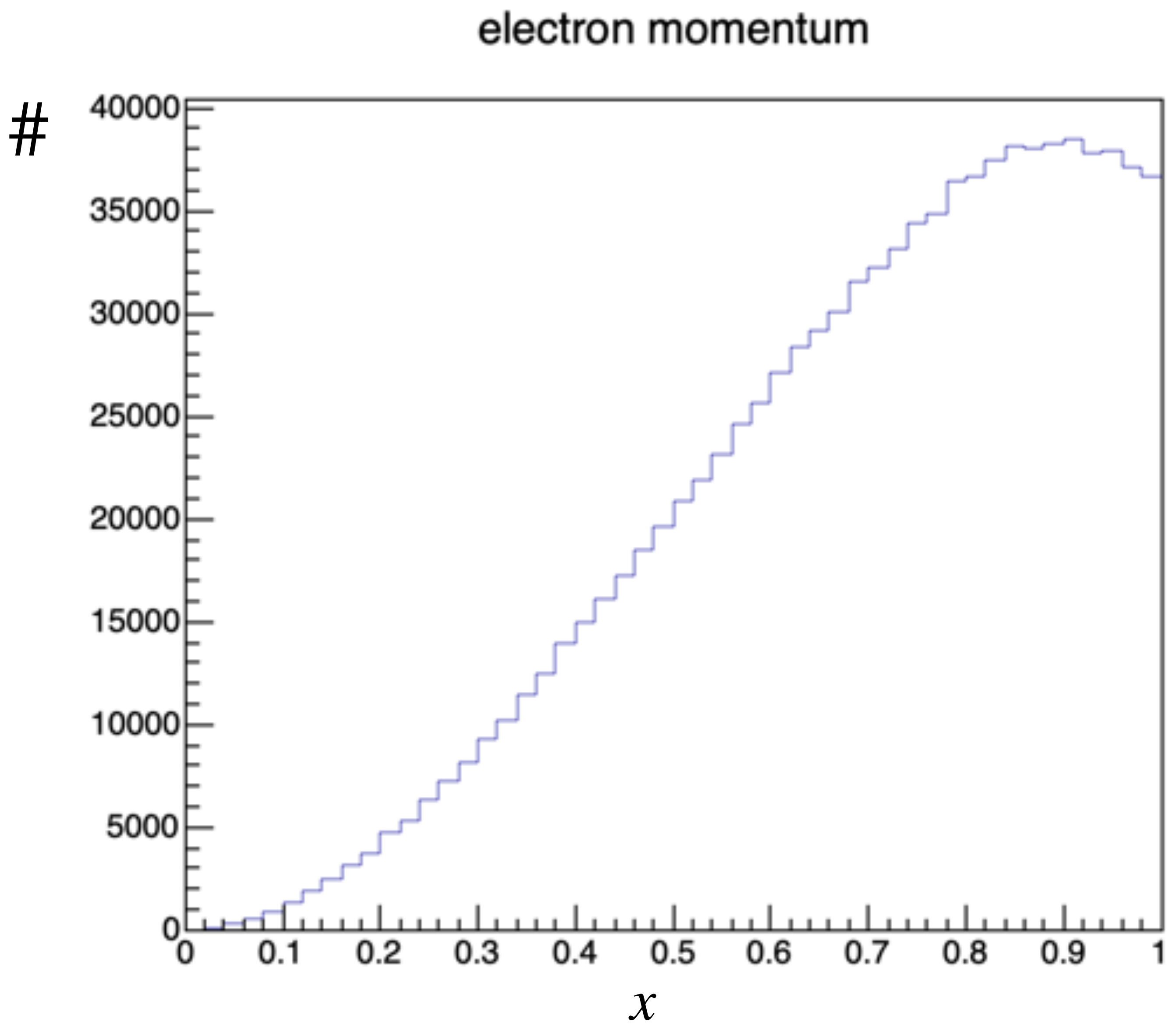
- The differential decay probability

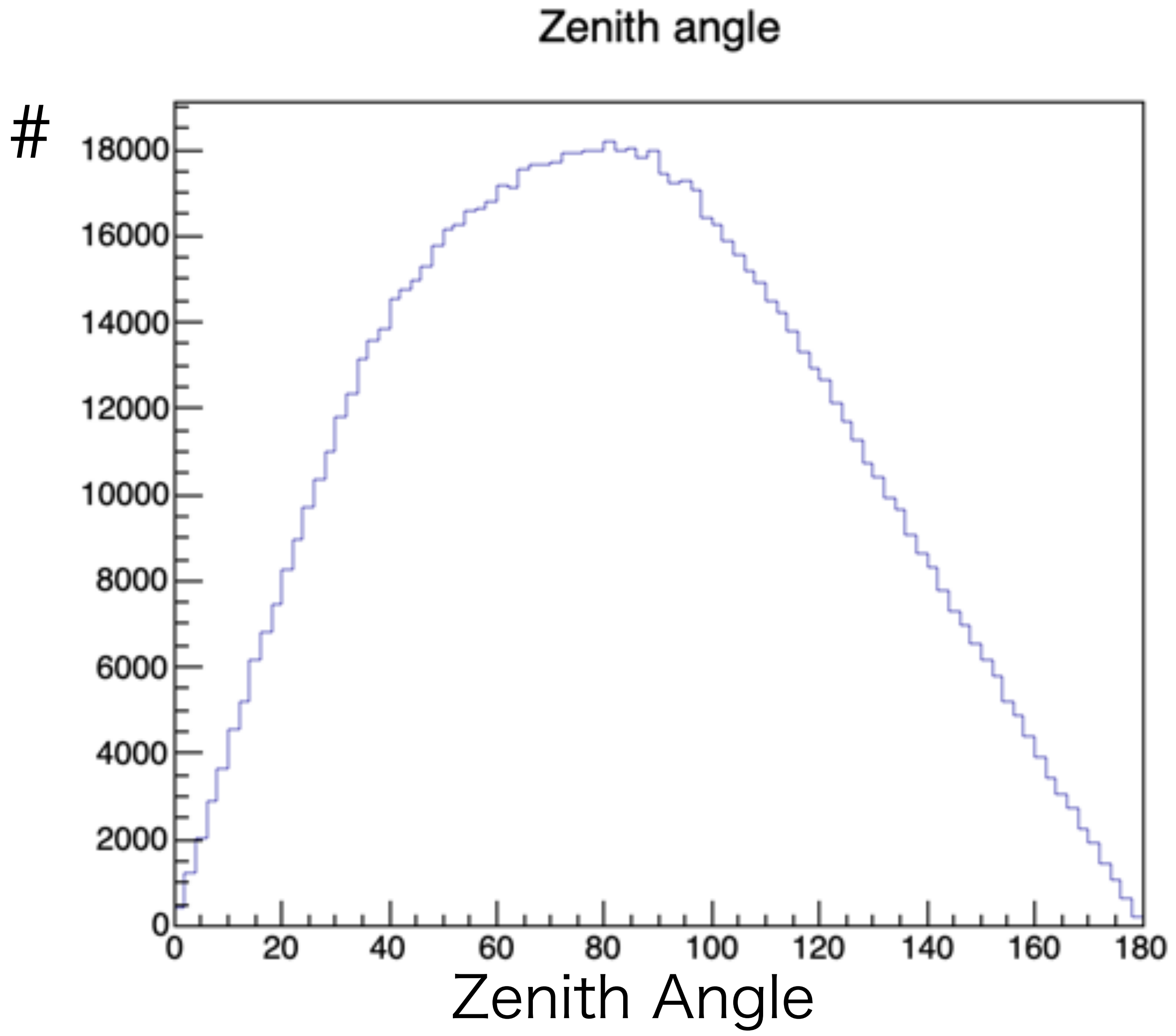


$$\frac{d^2\Gamma}{dxd\theta} \propto \{3 - 2x + cos\theta \cdot (2x - 1)\} \cdot sin\theta \cdot x^2$$

θ : angle between the electron momentum and muon spin

x : $E_e/max(E_e)$ where $max(E_e) = (m_\mu^2 + m_e^2)/2m_\mu$





Reliability of the Simulation

・To check the reliability,
compare the result of previous experiment and result from simulation

	Muon triggered	Entries of Electron
Result	500,000	100
Simulation	500,000	190

・Simulation yields around 2 times bigger number than previous one
But, it might be the result of the way of analyze

・If we can analyze without discarding the true signal,
we can trust this simulation

Performance evaluation of the detectors for measuring magnetic moment of muon

Itaru Kitano 2023/12/14

Motivation of the experiment

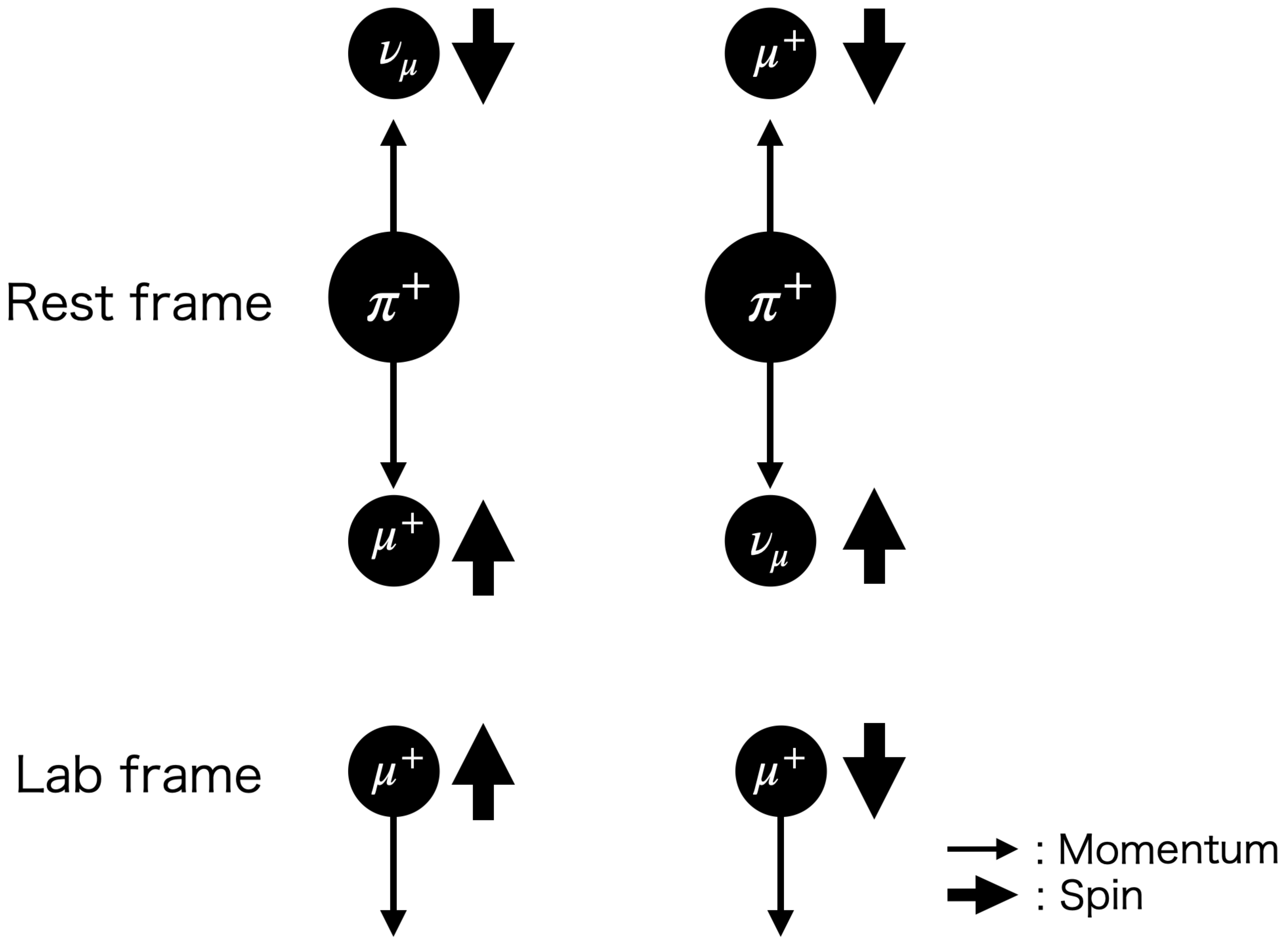
- I want to study something related to spin.
- This experiment is similar to what we did in first semester.

$$\overrightarrow{\mu} = -\frac{ge}{2m_\mu}\vec{s}$$

μ : magnetic moment

Occurrence of muon and its behavior in magnetic field

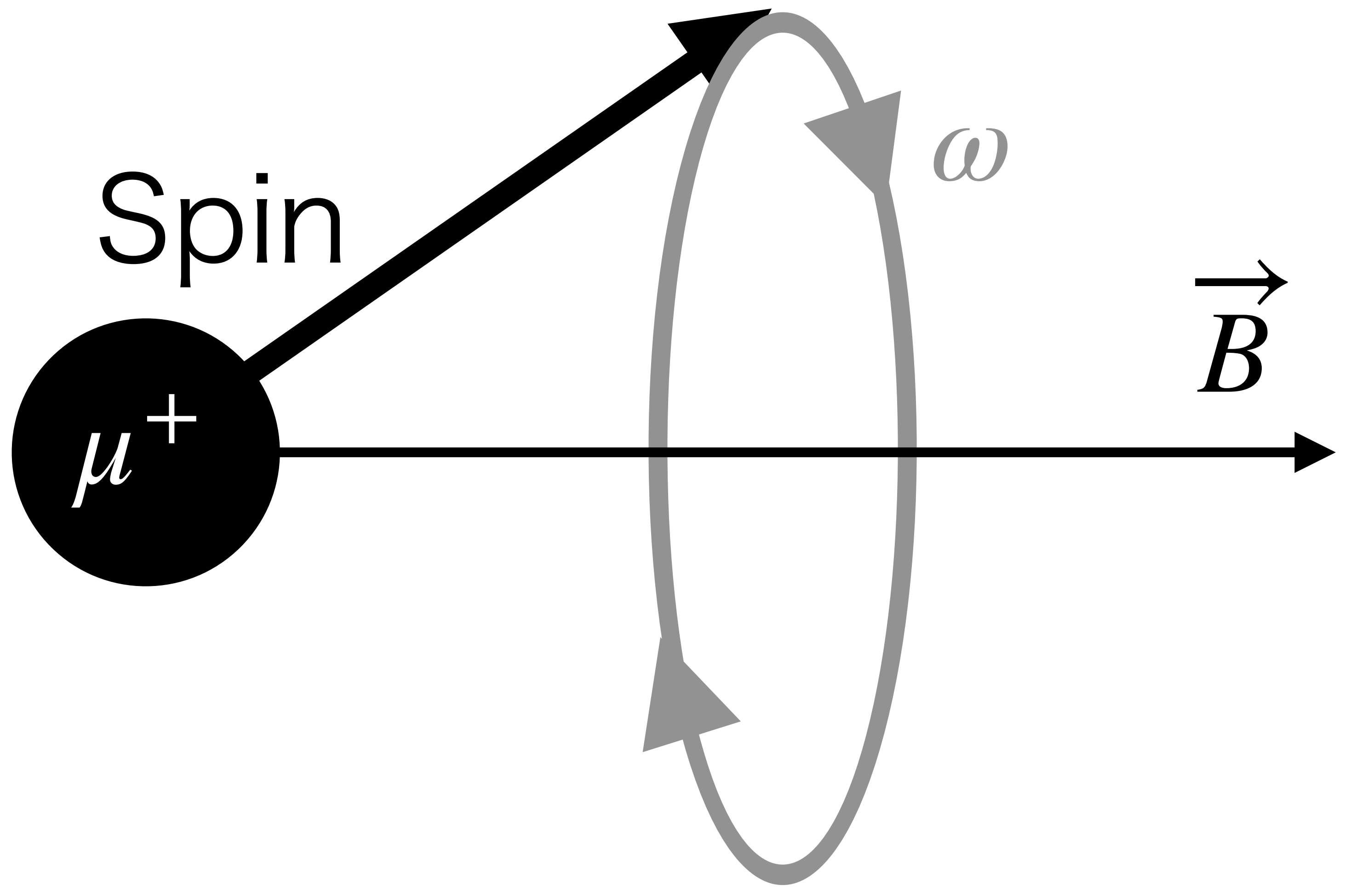
- Muon is generated by the decay of pion.
  - $\pi^+ \rightarrow \mu^+ + \nu_\mu$



Spin polarization of muon occurs.

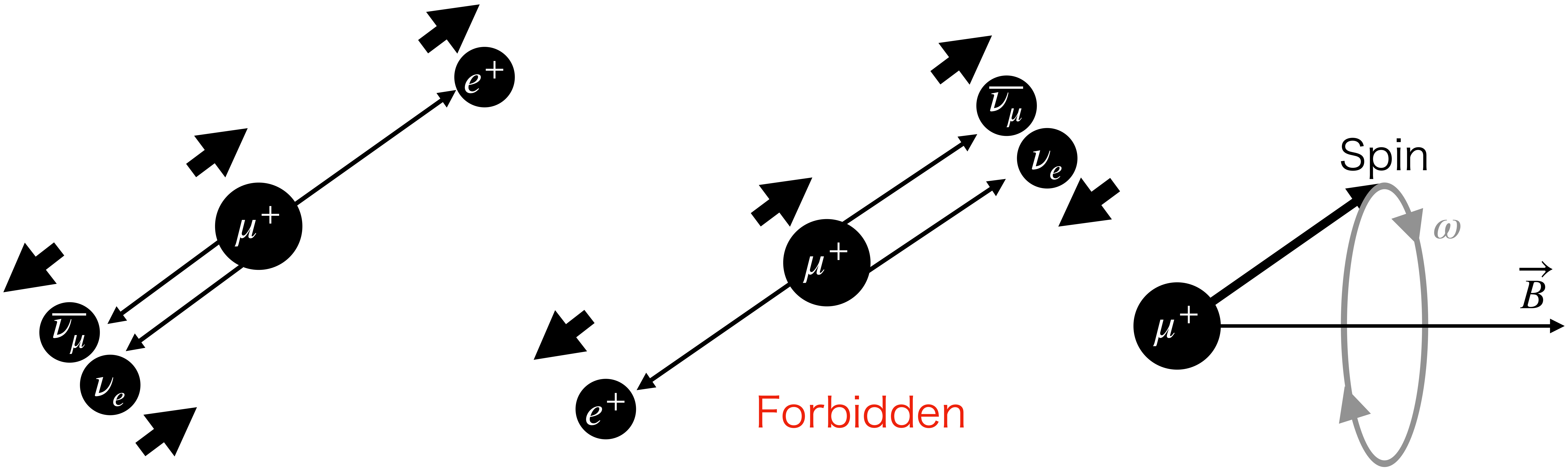
Spin in magnetic field precess with frequency ω.

$$\omega = \frac{geB}{2m_\mu}$$



Decay of muon

$\mu^+ \rightarrow e^+ + \overline{\nu_\mu} + \nu_e$

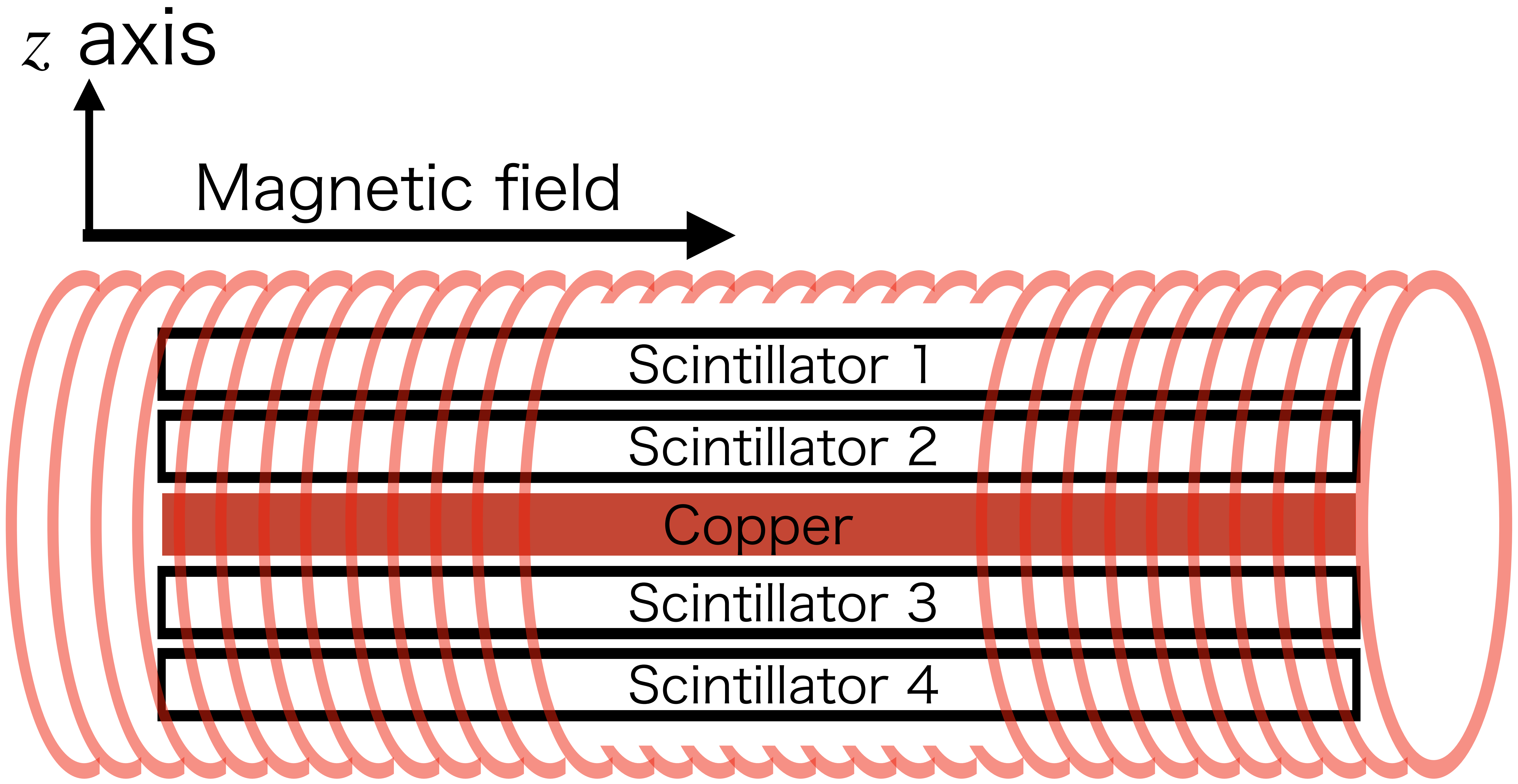


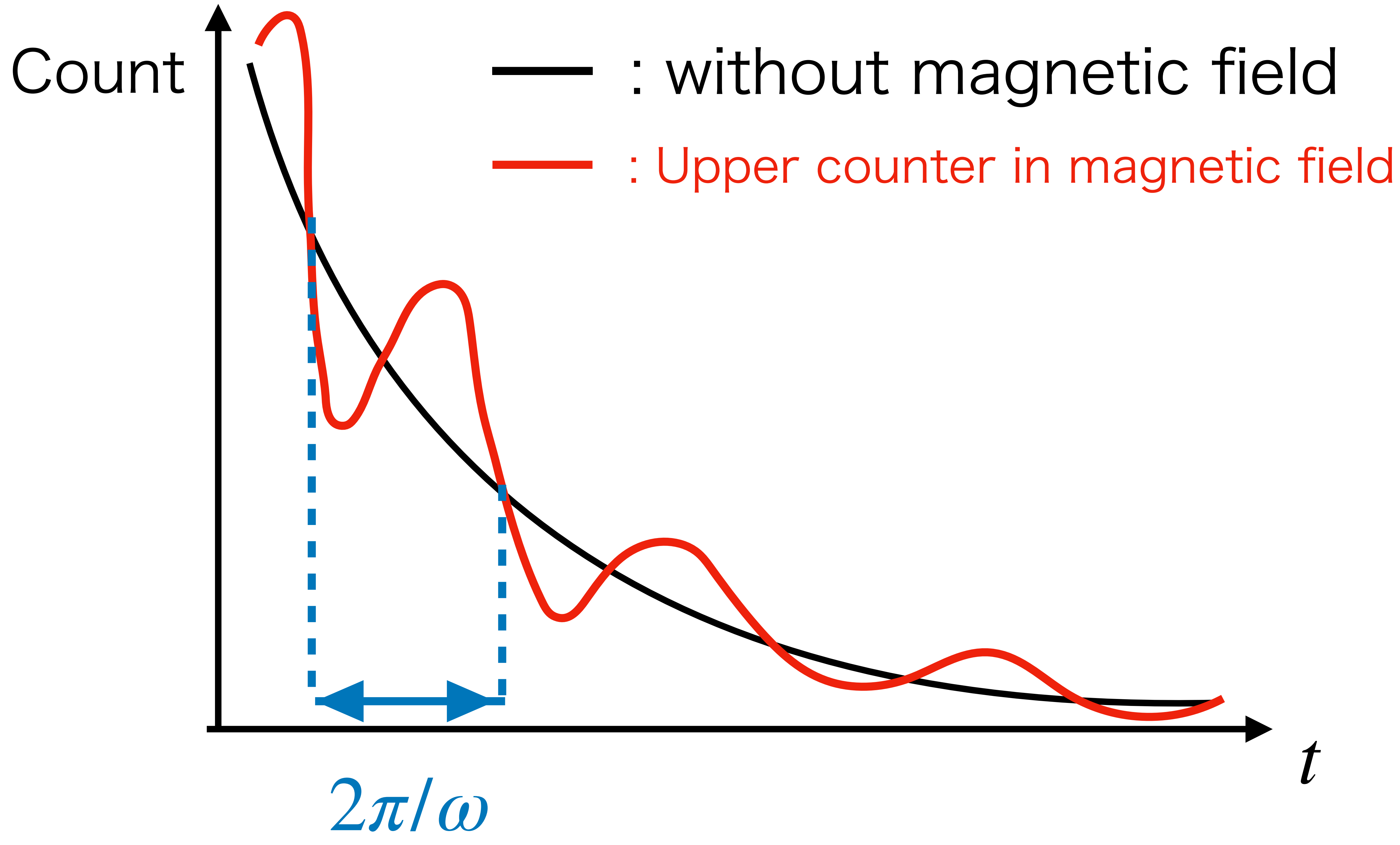
: Spin

: Momentum

The direction of outgoing positron is biased.

Observation of muon decay





The g factor can be obtained by observing the lifetime curve changed by the magnetic field.

Performance evaluation of PMTs

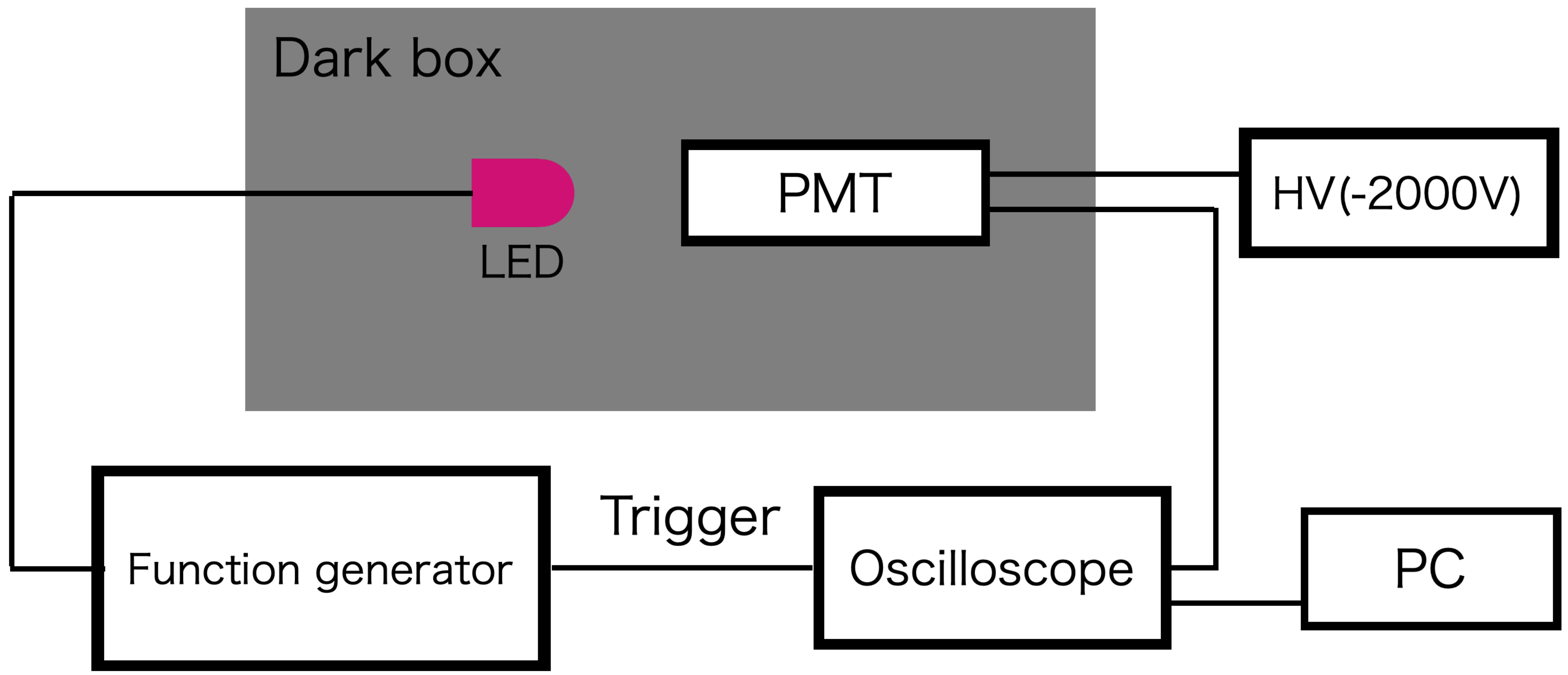
- Motivation to evaluate performance

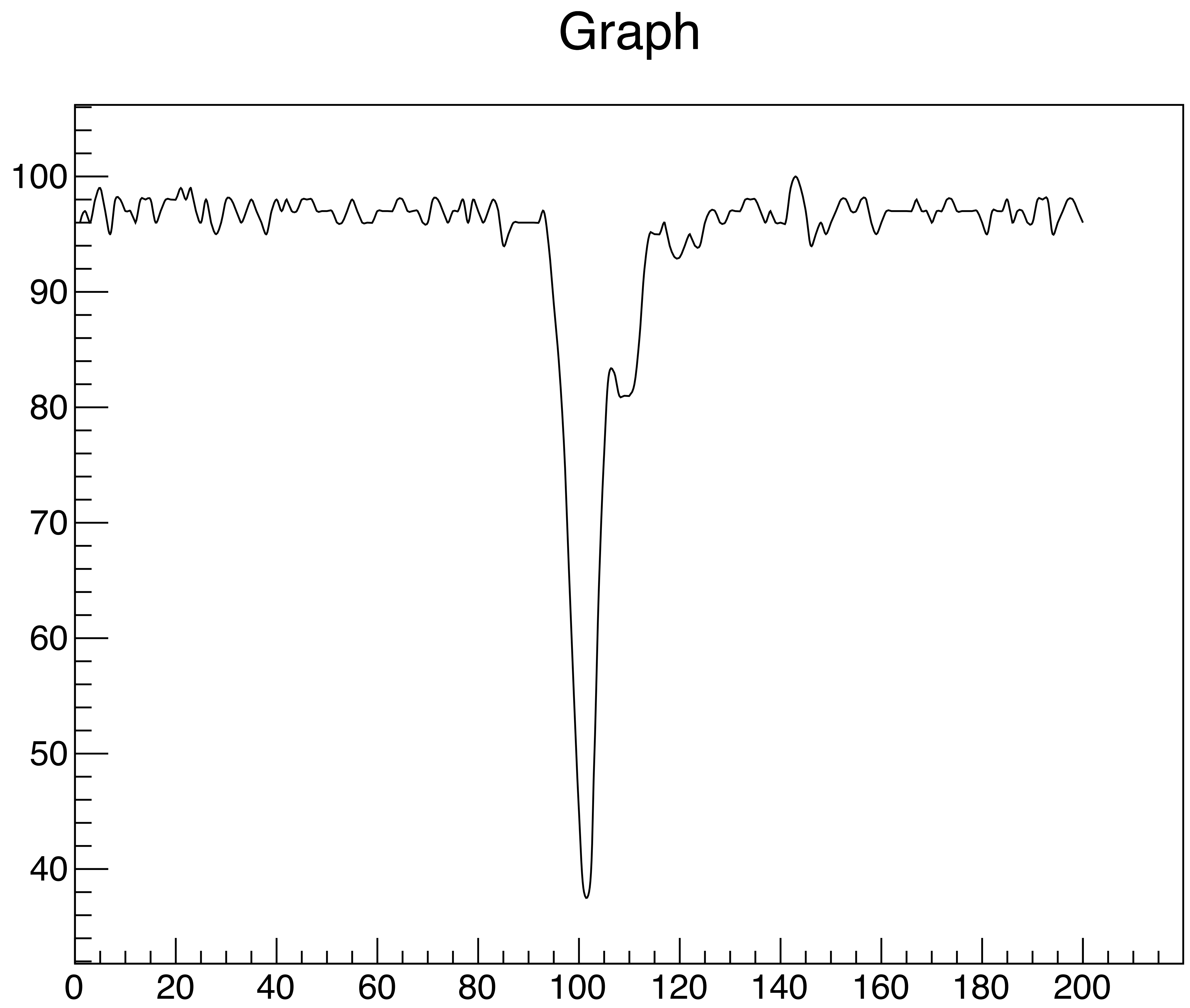
  →There are 12 PMTs in the lab, and I want to choose the best 6 of them.

- How did I evaluate performance of PMTs ?

  1. Photon counting to check the gain.
  2. Checking the noise rate.

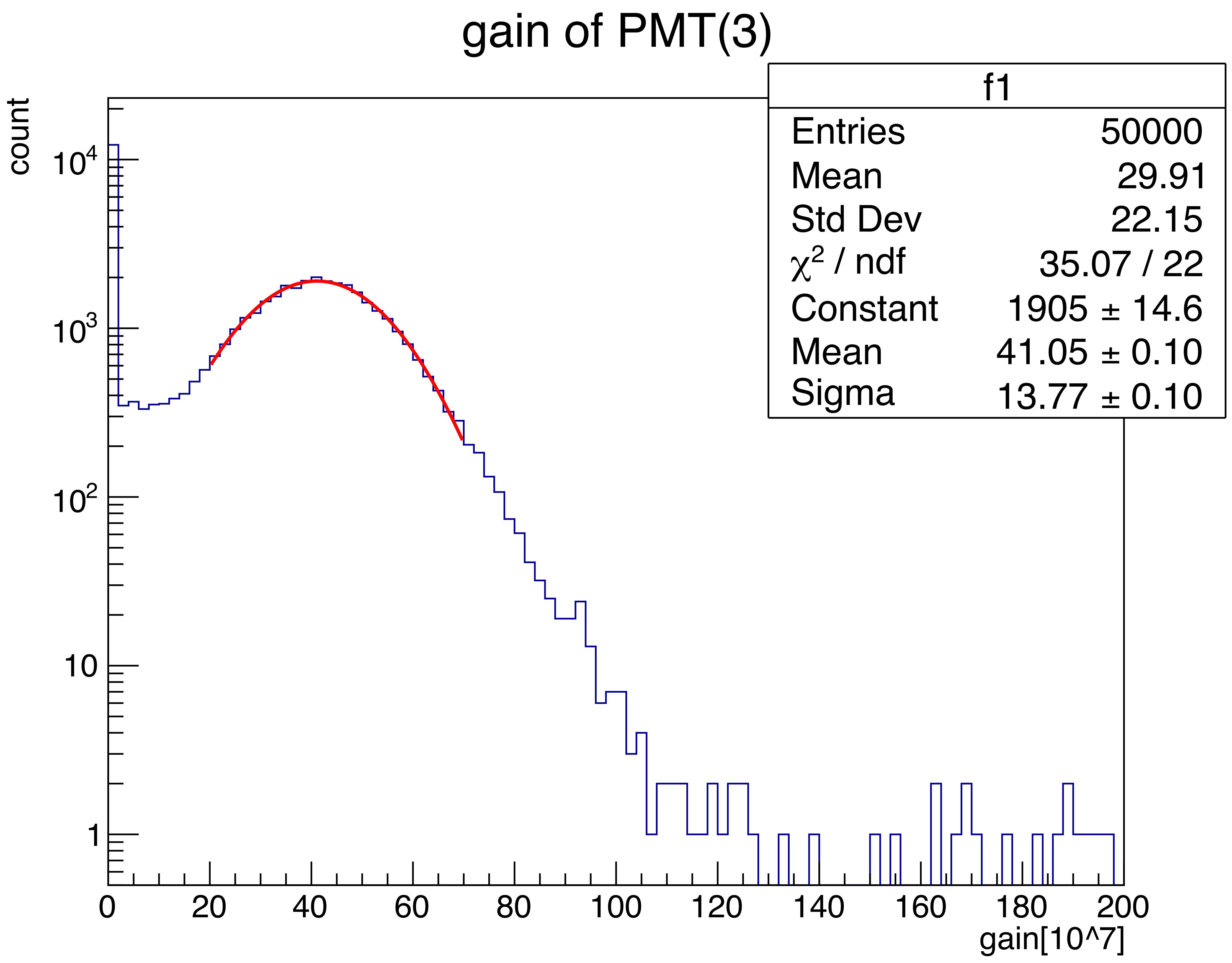
Photon counting

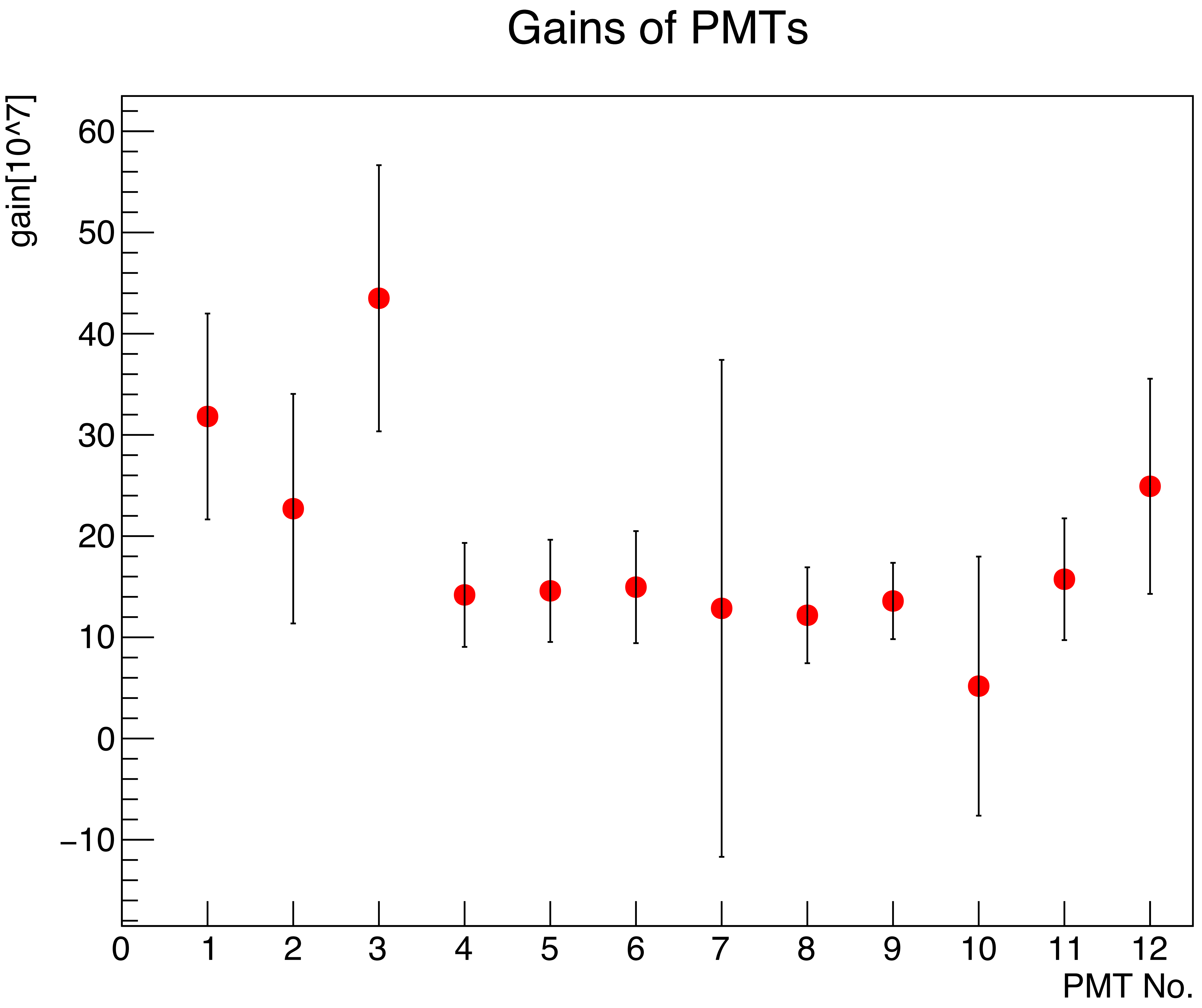




By converting voltage to current and integrating over time, gain can be obtained.

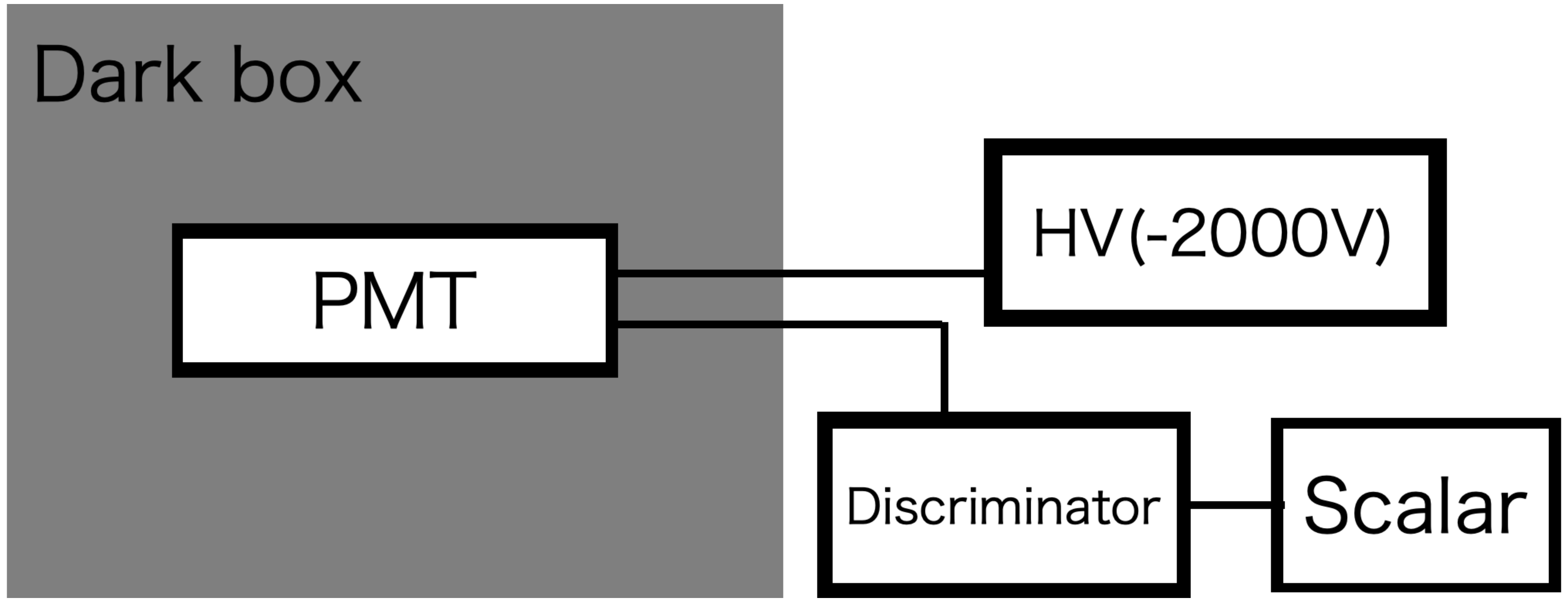
Gain

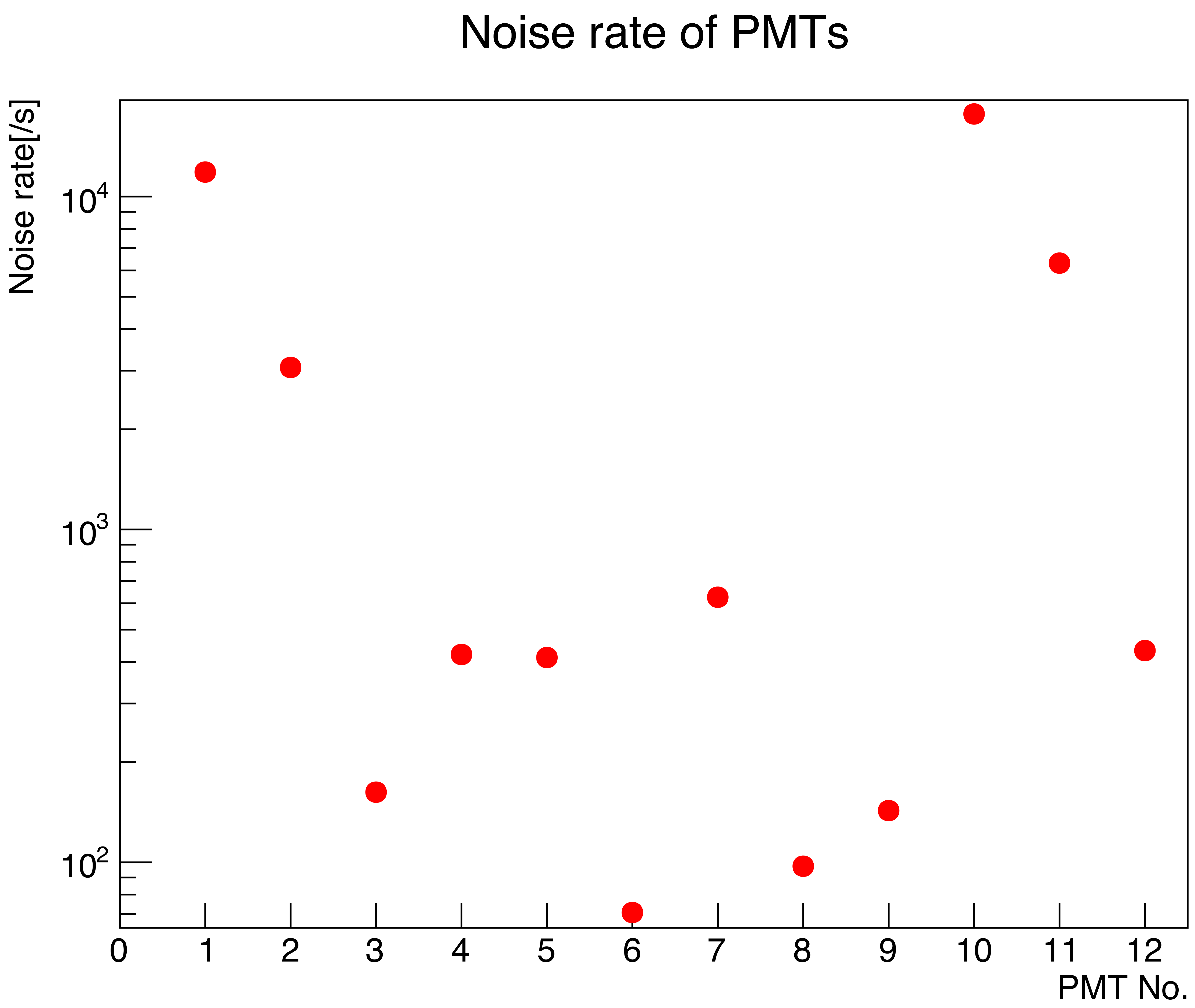

gain of PMT(3)
f1
Entries 50000
Mean 29.91
Std Dev 22.15
χ2 / ndf 35.07 / 22
Constant 1905 ± 14.6
Mean 41.05 ± 0.10
Sigma 13.77 ± 0.10
count
gain[10^7]


Gains of PMTs
gain[10^7]
PMT No.

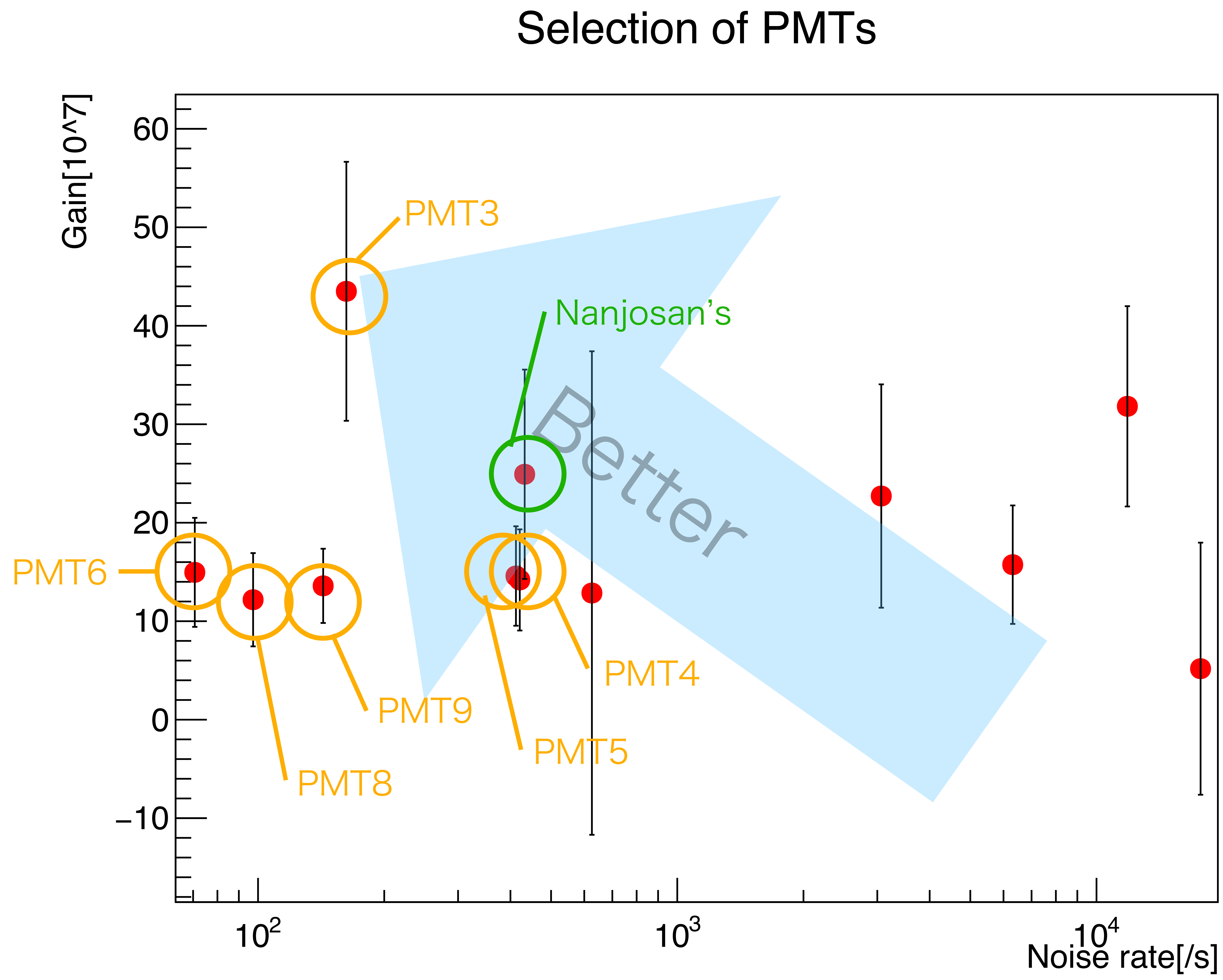
Rate of noise

- Threshold of discriminator is set to -23.7mV(minimum value).





Selection of PMTs

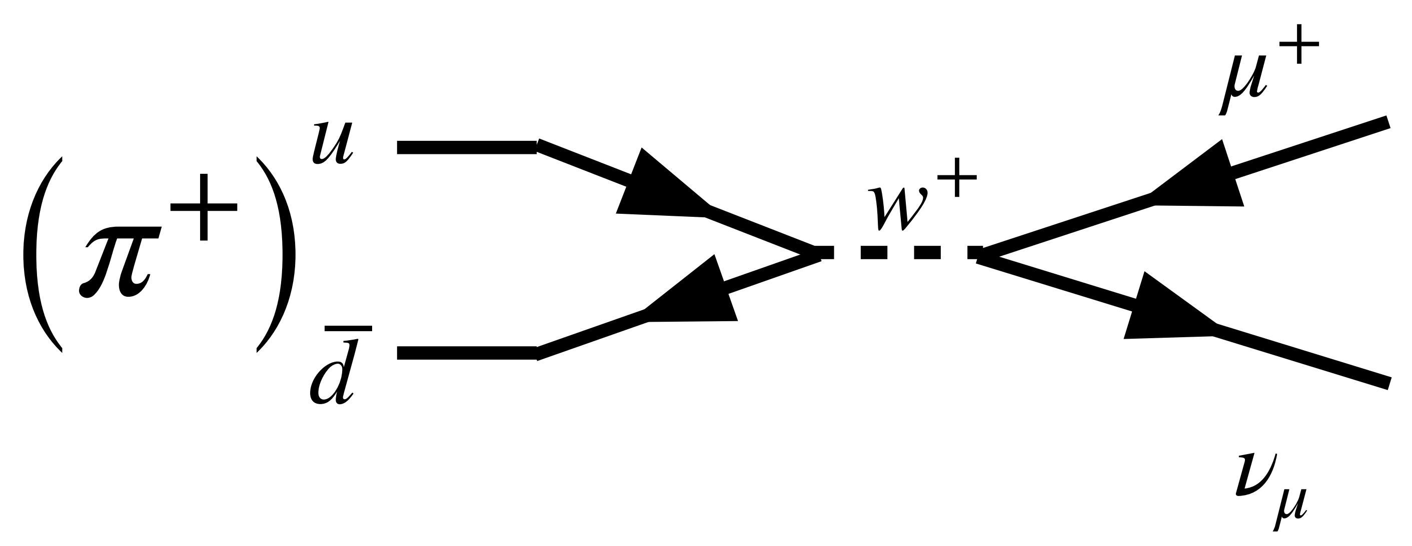


PMT 3, 4, 5, 6, 8, 9
are good to use!

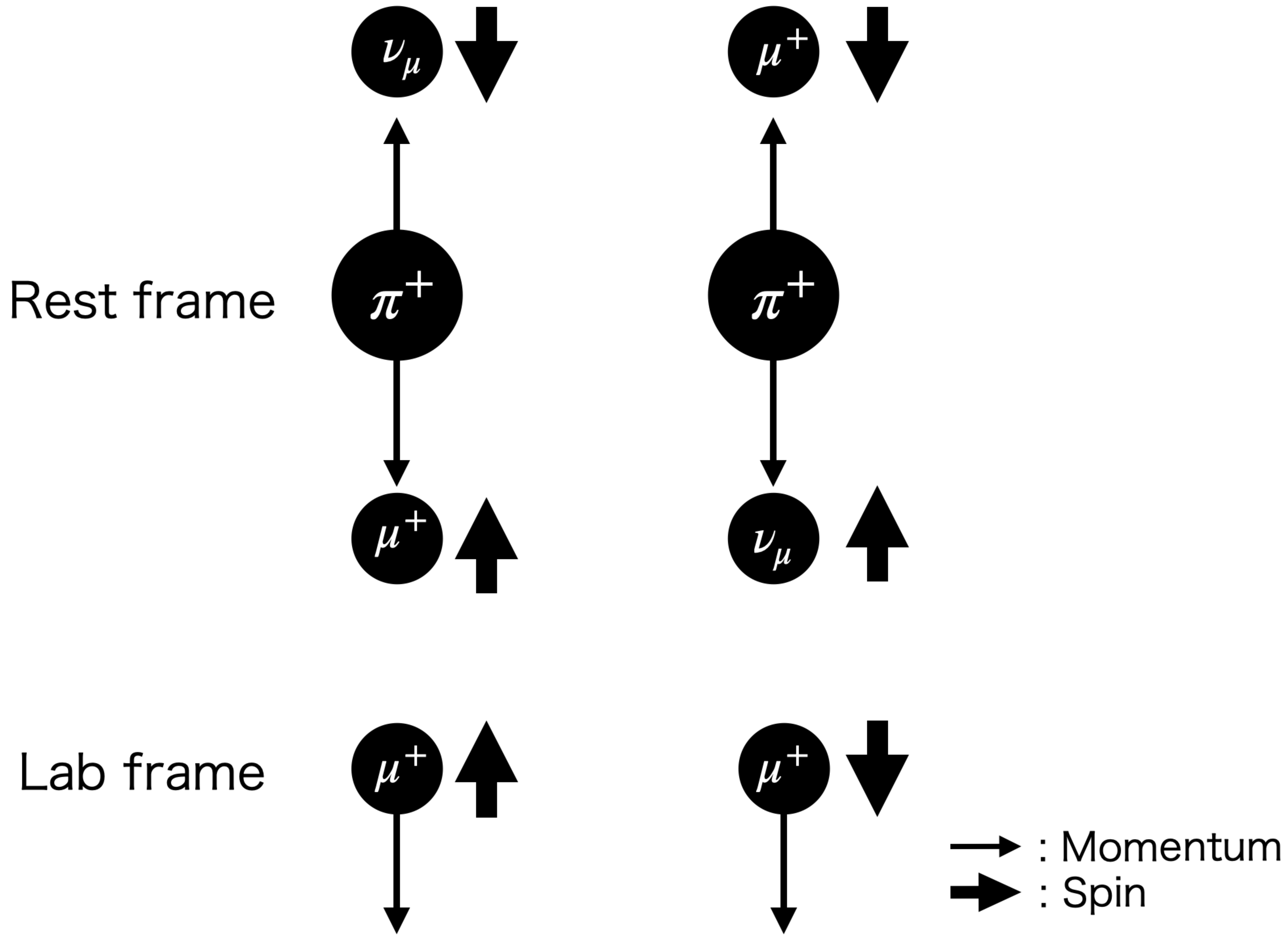
Conclusion

- I could choose the best 6 PMTs to use the experiment.
- Hereafter, I will mount the PMT on the light guide, and check the performance furthermore if I have time.

Decay of pion

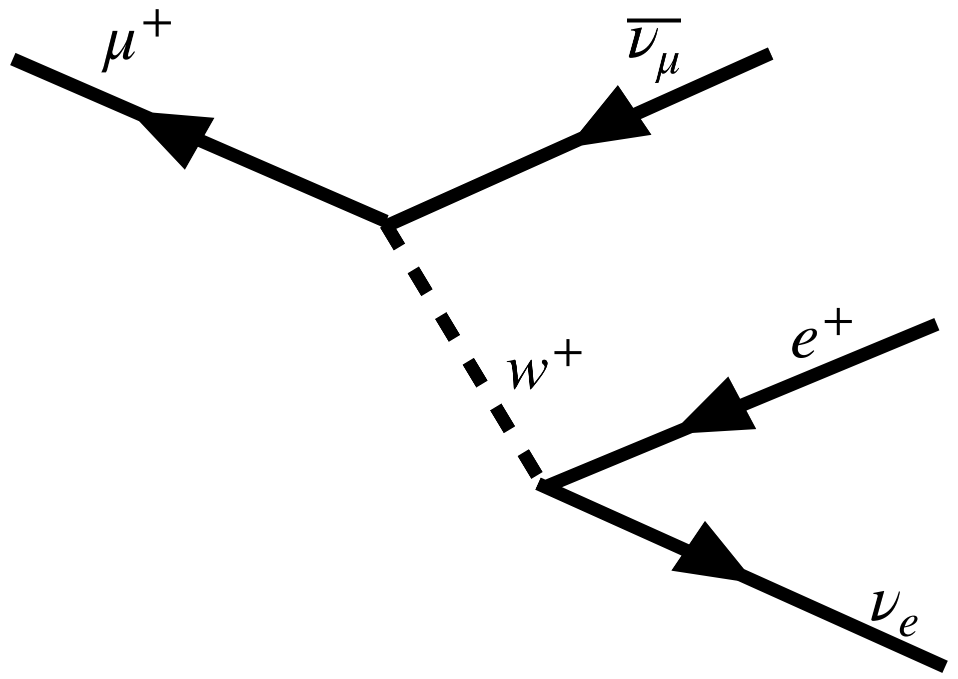


- Pion decays by weak interaction.
  - $\pi^+ \to \mu^+ + \nu_\mu$
- Because neutrino is relativistic, its helicity is left-handed. Then, from the conservation of angular momentum, decay of pion is as shown in the right figure.
- The energy of pion correspond to the same energy muon is different between left and right. The probability of existence of different energy states is different, thus muon spin polarization occurs.

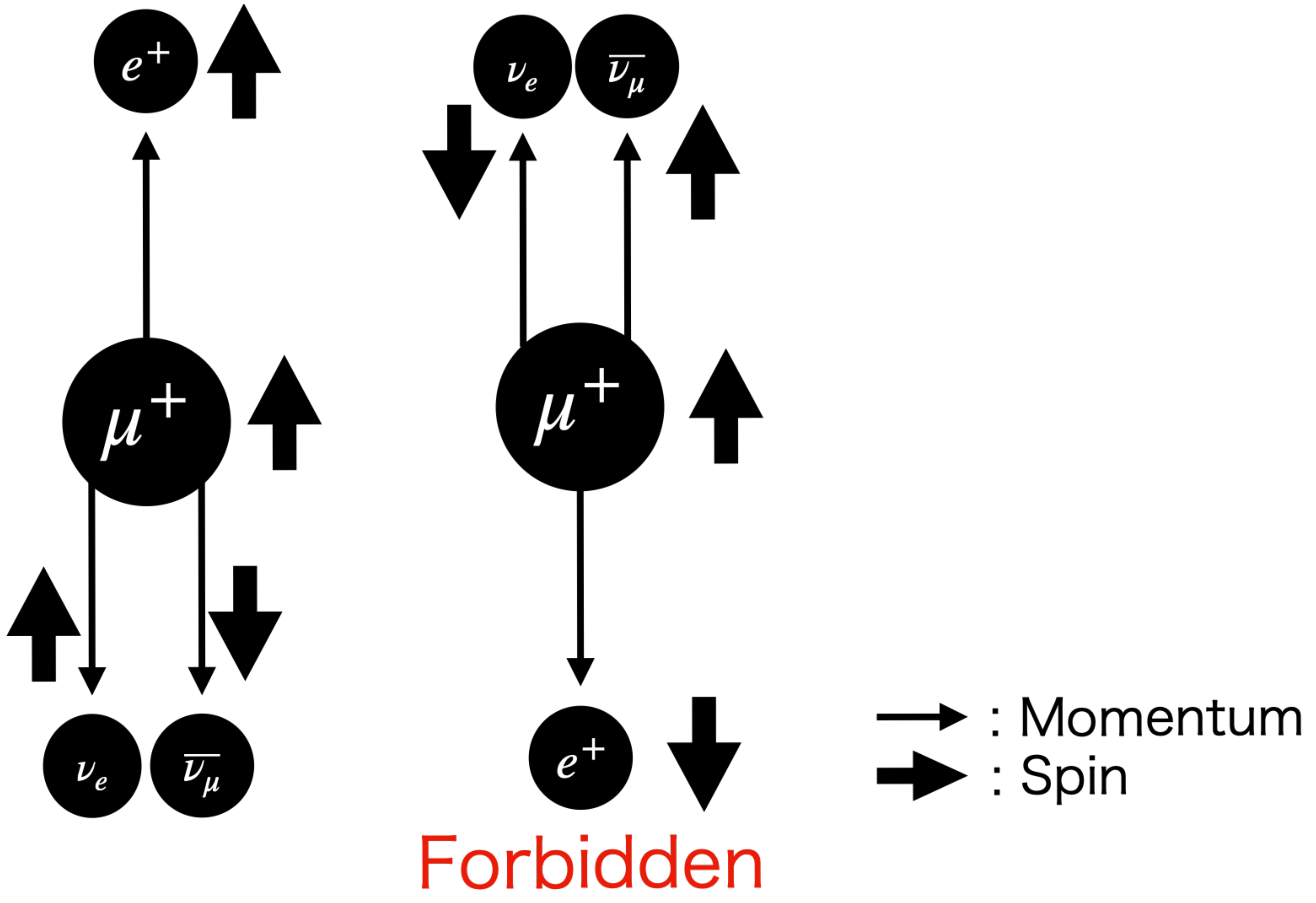


Decay of muon

- Muon decays by weak interaction.
  - $\mu^+ \rightarrow e^+ + \overline{\nu_\mu} + \nu_e$



- Weak interaction breaks parity symmetry, so the direction of the outgoing positron is biased.



Both of the above two figures show the situation where outgoing positron has the maximum energy. Particles are left-handed and antiparticles are right-handed, so the spin directions are as shown in the figures above. However, right one is forbidden because it doesn't conserve angular momentum.