Description of curved spacetime by matrix models

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Abstract

We study how curved spacetimes and topologically nontrivial configurations are realized by matrix models. First, we investigate relationship between a gauge theory on a principal bundle and that on its base space. In the case where the principal bundle is itself a group manifold, we also study relations of those gauge theories with a matrix model obtained by dimensionally reducing to zero dimensions. We show a relationship that Yang-Mills (YM) on a $U(1)$ and a $SU(2)$ bundle is equivalent to the theory around a certain background of YM-higgs on its base space. We also show that the theory around each monopole vacuum of YM-higgs on $SU(n+1)/(SU(n)\times U(1)) \simeq CP^n$ is equivalent to the theory around a certain vacuum of a matrix model in a commutative limit. By combining this with the relationship concerning a $U(1)$ bundle, we realize YM-higgs on $SU(n+1)/SU(n) \simeq S^{2n+1}$ in the matrix model.

We also apply these findings to other field theories; topological field theory and super Yang-Mills theory (SYM). In particular, we show the relations between Chern-Simons theory on $S^3(/Z_k)$, BF theory with a mass term on S^2, which is equivalent to the two-dimensional YM on S^2, and a matrix model and those between $\mathcal{N}=4$ SYM on $R\times S^3(/Z_k)$, 2+1 SYM on $R\times S^2$ and the plane wave matrix model (PWMM). In the former relation, we find that a certain sector of the matrix integration of the matrix model reproduces the partition function of YM on S^2. We also show that the latter relations are completely consistent with the gravity dual given by Lin and Maldacena, and we obtain a nontrivial evidence for the gauge/gravity correspondence. Furthermore, by using the latter relation, we propose a nonperturbative definition of $\mathcal{N}=4$ SYM on $R\times S^3$. We realize $\mathcal{N}=4$ SYM on $R\times S^3$ as the theory around a vacuum of PWMM. Our regularization preserves 16 supersymmetries and the gauge symmetry. We perform the one-loop calculation to give some evidence that in the continuum limit the superconformal symmetry is restored.